```
w-pred-aux  THM

⊢  ∀the_w:World, t:ℕ, i:Id. ¬first(<i,t>) ⇒ pred(<i,t>) ∈ E
| by InductionOnNat THEN RecUnfold ‘w-first‘ 0
|    THEN
|    RecUnfold ‘w-pred‘ 0
|    THEN
|    Unfolds [‘w-loc‘;‘w-time‘] 0
|    THEN
|    Reduce 0
|    THEN
|    Try (Complete (Auto THEN Analyze -1))
|    THEN
|    SplitOnConclITE
|
|\
| .....truecase .....
| 1. the_w : World
| 2. t : ℤ
| 3. 0<t
| 4. ∀i:Id. ¬first(<i,t-1>) ⇒ pred(<i,t-1>) ∈ E
| 5. t = 0
| ⊢  ∀i:Id.
|    ¬true₂ ⇒ if isnull(a(i;t-1))→ pred(<i,t-1>) else <i,t-1> fi ∈ E
|   by Auto
|
.\
. .....falsecase .....
. 1. the_w : World
. 2. t : ℤ
. 3. 0<t
. 4. ∀i:Id. ¬first(<i,t-1>) ⇒ pred(<i,t-1>) ∈ E
. 5. ¬t=₂0
. ⊢  ∀i:Id.
. |  ¬if isnull(a(i;t-1))→ first(<i,t-1>) else false₂ fi
. |  ⇒
. |  if isnull(a(i;t-1))→ pred(<i,t-1>) else <i,t-1> fi ∈ E
. | by (Analyze 0) THEN SplitOnConclITE
. |
. |\
. | .....truecase .....
. | 1. the_w : World
. | 2. t : ℤ
. | 3. 0<t
. | 4. ∀i:Id. ¬first(<i,t-1>) ⇒ pred(<i,t-1>) ∈ E
. | 5. ¬t=₂0
. | 6. i : Id
. | 7. isnull(a(i;t-1))
. | ⊢  ¬first(<i,t-1>) ⇒ pred(<i,t-1>) ∈ E  by SimpleInstHyp i 4
. |
. .\
. . .....falsecase .....
. . 1. the_w : World
. . 2. t : ℤ
. . 3. 0<t
```

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. . 4. ∀i:Id. ¬first(<i,t-1>) ⇒ pred(<i,t-1>) ∈ E
. . 5. ¬t=₂0
. . 6. i : Id
. . 7. ¬isnull(a(i;t-1))
. . ⊢  ¬false₂ ⇒ <i,t-1> ∈ E
. .   by Unfold `w-E` 0 THEN Analyze THEN Reduce 0
END
```