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w-withlnk_wf  THM

⊢  ∀the_w:World, l:IdLnk, mss:Msg List.
|  withlnk(l;mss) ∈ (t:Id×the_w.M(l,t)) List  by All_Wld
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| .....subterm .....
| 1. T : Id→Id→Type
| 2. TA : Id→Id→Type
| 3. M : IdLnk→Id→Type
| 4. i:Id→ℕ→(x:Id→T(i,x))
| 5. i:Id→ℕ→action(w-action-dec(TA;M;i))
| 6. i:Id→ℕ→({m:Msg(M)| source(mlnk(m)) = i } List)
| 7. Top
| 8. l : IdLnk
| 9. Msg(M) List
| 10. ms : {x:Msg(M)| (λms.mlnk(ms) = l)(x) }
| ⊢  2of(ms) ∈ t:Id×M(l,t)
| | by Analyze -1 THEN Analyze -2 THEN Unfold `mlnk` -1
| |    THEN
| |    All Reduce
| |
| 1. T : Id→Id→Type
| 2. TA : Id→Id→Type
| 3. M : IdLnk→Id→Type
| 4. i:Id→ℕ→(x:Id→T(i,x))
| 5. i:Id→ℕ→action(w-action-dec(TA;M;i))
| 6. i:Id→ℕ→({m:Msg(M)| source(mlnk(m)) = i } List)
| 7. Top
| 8. l : IdLnk
| 9. Msg(M) List
| 10. l1 : IdLnk
| 11. m1 : t:Id×M(l1,t)
| 12. l1 = l
| ⊢  m1 ∈ t:Id×M(l,t)  by Analyze -2 THEN Reduce 0 THEN Analyze
| |
| .....subterm .....
| 1. T : Id→Id→Type
| 2. TA : Id→Id→Type
| 3. M : IdLnk→Id→Type
| 4. i:Id→ℕ→(x:Id→T(i,x))
| 5. i:Id→ℕ→action(w-action-dec(TA;M;i))
| 6. i:Id→ℕ→({m:Msg(M)| source(mlnk(m)) = i } List)
| 7. Top
| 8. l : IdLnk
| 9. Msg(M) List
| 10. l1 : IdLnk
| 11. t : Id
| 12. m2 : M(l1,t)
| 13. l1 = l
| ⊢  m2 ∈ M(l,t)  by Assert (M(l1,t) = M(l,t))
| |
| .....assertion .....
| 1. T : Id→Id→Type
| 2. TA : Id→Id→Type
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| 3. M : IdLnk→Id→Type
| 4. i:Id→ℕ→(x:Id→T(i,x))
| 5. i:Id→ℕ→action(w-action-dec(TA;M;i))
| 6. i:Id→ℕ→({m:Msg(M)| source(mlnk(m)) = i } List)
| 7. Top
| 8. l : IdLnk
| 9. Msg(M) List
| 10. l1 : IdLnk
| 11. t : Id
| 12. M(l1,t)
| 13. l1 = l
| ⊢  M(l1,t) = M(l,t)  by Analyze THEN Analyze
| |
| .....subterm .....
| 1. T : Id→Id→Type
| 2. TA : Id→Id→Type
| 3. M : IdLnk→Id→Type
| 4. i:Id→ℕ→(x:Id→T(i,x))
| 5. i:Id→ℕ→action(w-action-dec(TA;M;i))
| 6. i:Id→ℕ→({m:Msg(M)| source(mlnk(m)) = i } List)
| 7. Top
| 8. l : IdLnk
| 9. Msg(M) List
| 10. l1 : IdLnk
| 11. t : Id
| 12. M(l1,t)
| 13. l1 = l
| ⊢  l1 = l
|   by RWO
|      Thm* ∀a,b:IdLnk. a = b ⟺ a = b    {assert_DASH_eq_DASH_lnk}
|      -1
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. .....subterm .....
. 1. T : Id→Id→Type
. 2. TA : Id→Id→Type
. 3. M : IdLnk→Id→Type
. 4. i:Id→ℕ→(x:Id→T(i,x))
. 5. i:Id→ℕ→action(w-action-dec(TA;M;i))
. 6. i:Id→ℕ→({m:Msg(M)| source(mlnk(m)) = i } List)
. 7. Top
. 8. l : IdLnk
. 9. Msg(M) List
. 10. x : Msg(M)
. ⊢  (λms.mlnk(ms) = l)(x) ∈ 𝔹  by Reduce 0
END
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