```
fpf-all-single-decl  THM

⊢  ∀A:Type, eq:EqDecider(A), P:(A→Type→Prop), x:A, v:Type.
|  ∀y∈dom(x : v). w=x : v(y) ⇒  P(y,w) ⟺ P(x,v)
| by Assert
|    (∀A:Type, eq:EqDecider(A), B:(A→Type), P:(x:A→B(x)→Prop), x:A,
|      v:B(x). ∀y∈dom(x : v). w=x : v(y) ⇒  P(y,w) ⟺ P(x,v))
|
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| .....assertion .....
| ⊢  ∀A:Type, eq:EqDecider(A), B:(A→Type), P:(x:A→B(x)→Prop), x:A,
| |   v:B(x). ∀y∈dom(x : v). w=x : v(y) ⇒  P(y,w) ⟺ P(x,v)
| | by UnivCD THEN Unfold `fpf-all` 0
| |    THEN
| |    Unfolds [`fpf-dom`;`fpf-single`;`fpf-ap`] 0
| |    THEN
| |    Reduce 0
| |
| 1. A : Type
| 2. eq : EqDecider(A)
| 3. B : A→Type
| 4. P : x:A→B(x)→Prop
| 5. x : A
| 6. v : B(x)
| ⊢  (∀y:A. (eqof(eq)(x,y) ∨₂ false₂) ⇒ P(y,v)) ⟺ P(x,v)  by Auto
| |
| |\
| | 1. A : Type
| | 2. eq : EqDecider(A)
| | 3. B : A→Type
| | 4. P : x:A→B(x)→Prop
| | 5. x : A
| | 6. v : B(x)
| | 7. ∀y:A. (eqof(eq)(x,y) ∨₂ false₂) ⇒ P(y,v)
| | ⊢  P(x,v)
| |   by BackThru -1
| |      THEN
| |      RWO
| |      Thm* ∀d:EqDecider(A), i:A. (eqof(d)(i,i)) ~ true₂
| |      {eqof_eq_btrue}
| |      0
| |      THEN
| |      Reduce 0
| |
| |\
| | .....unproved Inclusion subgoal .....
| | 1. A : Type
| | 2. eq : EqDecider(A)
| | 3. B : A→Type
| | 4. x:A→B(x)→Prop
| | 5. x : A
| | 6. v : B(x)
| | 7. y : A
| | 8. eqof(eq)(x,y) ∨₂ false₂
| | ⊢  v ∈ B(y)
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| | | by RW assert_pushdownC -1 THEN Analyze -1
| | |    THEN
| | |    RWO "deq_property<" -1
| | |
| | 1. A : Type
| | 2. EqDecider(A)
| | 3. B : A→Type
| | 4. x:A→B(x)→Prop
| | 5. x : A
| | 6. v : B(x)
| | 7. y : A
| | 8. x = y
| | ⊢  v ∈ B(y)  by SubsumeC (B(x))
| | |
| | |\
| | | 1. A : Type
| | | 2. EqDecider(A)
| | | 3. B : A→Type
| | | 4. x:A→B(x)→Prop
| | | 5. x : A
| | | 6. v : B(x)
| | | 7. y : A
| | | 8. x = y
| | | ⊢  v ∈ B(x)  by Auto
| | |
| | .\
| | . .....subtype_rel .....
| | . 1. A : Type
| | . 2. EqDecider(A)
| | . 3. B : A→Type
| | . 4. x:A→B(x)→Prop
| | . 5. x : A
| | . 6. B(x)
| | . 7. y : A
| | . 8. x = y
| | . ⊢  (B(x)) ⊆r (B(y))  by RevHypSubst -1 0
| |
| .\
| . 1. A : Type
| . 2. eq : EqDecider(A)
| . 3. B : A→Type
| . 4. P : x:A→B(x)→Prop
| . 5. x : A
| . 6. v : B(x)
| . 7. P(x,v)
| . 8. y : A
| . 9. eqof(eq)(x,y) ∨2 false2
| . ⊢  P(y,v)
| . | by RW assert_pushdownC -1 THEN Analyze -1
| . |    THEN
| . |    RWO "deq_property<" -1
| . |
| . 1. A : Type
| . 2. EqDecider(A)
| . 3. B : A→Type
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| . 4. P : x:A→B(x)→Prop
| . 5. x : A
| . 6. v : B(x)
| . 7. P(x,v)
| . 8. y : A
| . 9. x = y
| . ⊢  P(y,v)  by StrongRevHypSubst -1 0
| . |
| . .....unproved Inclusion subgoal .....
| . 1. A : Type
| . 2. EqDecider(A)
| . 3. B : A→Type
| . 4. P : x:A→B(x)→Prop
| . 5. x : A
| . 6. v : B(x)
| . 7. P(x,v)
| . 8. y : A
| . 9. x = y
| . 10. z : A
| . 11. z = x
| . ⊢  v ∈ B(z)  by DoSubsume
| . |
| . .....subtype_rel .....
| . 1. A : Type
| . 2. EqDecider(A)
| . 3. B : A→Type
| . 4. P : x:A→B(x)→Prop
| . 5. x : A
| . 6. v : B(x)
| . 7. P(x,v)
| . 8. y : A
| . 9. x = y
| . 10. z : A
| . 11. z = x
| . ⊢  (B(x)) ⊆r (B(z))  by HypSubst -1 0
|
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. 1. ∀A:Type, eq:EqDecider(A), B:(A→Type), P:(x:A→B(x)→Prop), x:A,
.     v:B(x). ∀y∈dom(x : v). w=x : v(y) ⇒  P(y,w) ⟺ P(x,v)
. ⊢  ∀A:Type, eq:EqDecider(A), P:(A→Type→Prop), x:A, v:Type.
.    ∀y∈dom(x : v). w=x : v(y) ⇒  P(y,w) ⟺ P(x,v)
.   by RepeatFor 2 (ParallelOp -1) THEN InstHyp [λ₂x. Type] -1
END
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