```
fpf-join-is-empty  THM

⊢  ∀A:Type, eq:EqDecider(A), f,g:x:A fp-> Top.
|  fpf-is-empty(f ⊕ g) ~ (fpf-is-empty(f)∧₂fpf-is-empty(g))
| by Auto THEN Analyze -2 THEN Analyze -1
|    THEN
|    Repeat
|    (Unfolds [‘fpf-is-empty‘;‘fpf-join‘;‘fpf-dom‘] 0 THEN Reduce 0)
|    THEN
|    All Thin
|
1. A : Type
2. eq : EqDecider(A)
3. d : A List
4. d1 : A List
⊢  (||d @ filter(λa.¬₂deq-member(eq;a;d);d1)||=₂0)
|  =
|  ((||d||=₂0)∧₂(||d1||=₂0))  by Analyze -2 THEN Reduce 0
|
|\
| 1. A : Type
| 2. EqDecider(A)
| 3. d1 : A List
| ⊢  (||filter(λa.true₂;d1)||=₂0) = (||d1||=₂0)
| | by RWO
| |    Thm* ∀P:(T→𝔹), L:T List. (∀x∈L.P(x)) ⇒ (filter(P;L) ~ L)
| |    {filter_trivial}
| |    0
| |
| 1. A : Type
| 2. EqDecider(A)
| 3. d1 : A List
| ⊢  (∀x∈d1.(λa.true₂)(x))
|   by Unfold ‘l_all‘ 0 THEN ReduceSOAps 0 THEN Reduce 0
|
.\
. 1. A : Type
. 2. eq : EqDecider(A)
. 3. u : A
. 4. v : A List
. 5. d1 : A List
. ⊢  (||v @ filter(λa.¬₂(eqof(eq)(u,a) ∨₂ deq-member(eq;a;v));d1)||
.     +1=₂0)
.    =
.    ((||v||+1=₂0)∧₂(||d1||=₂0))
.   by RWO
.      Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
.      {length_append}
.      0
.      THEN
.      AutoBoolCase (||v||+1=₂0)
.      THEN
.      AutoBoolCase
.      (||v||+||filter(λa.
.       ¬₂(eqof(eq)(u,a) ∨₂ deq-member(eq;a;v));d1)||+1=₂0)
```

```
END
```