```
fpf-join_wf  THM

⊢  ∀A:Type, B:(A→Type), f,g:a:A fp-> B(a), eq:EqDecider(A).
|  f ⊕ g ∈ a:A fp-> B(a)
| by Unfolds [‘fpf‘;‘fpf-join‘] 0 THEN Try (Unfold ‘fpf-dom‘ 0)
|    THEN
|    All (λi.Fold ‘fpf‘ i THEN Complete Auto)
|    THEN
|    AssertBY (f ∈ a:A fp-> B(a))
|    (All (λi.Fold ‘fpf‘ i THEN Complete Auto))
|    THEN
|    AssertBY (g ∈ a:A fp-> B(a))
|    (All (λi.Fold ‘fpf‘ i THEN Complete Auto))
|
1. A : Type
2. B : A→Type
3. f : d:A List×a:{a:A| (a ∈ d) }→B(a)
4. g : d:A List×a:{a:A| (a ∈ d) }→B(a)
5. eq : EqDecider(A)
6. a : {a:A| (a ∈ 1of(f) @ filter(λa.¬₂a ∈ dom(f);1of(g))) }
7. f ∈ a:A fp-> B(a)
8. g ∈ a:A fp-> B(a)
⊢  f(a)?g(a) ∈ B(a)
| by Analyze -3 THEN Unfold ‘fpf-cap‘ 0 THEN SplitOnConclITE
|
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| .....truecase .....
| 1. A : Type
| 2. B : A→Type
| 3. f : d:A List×a:{a:A| (a ∈ d) }→B(a)
| 4. g : d:A List×a:{a:A| (a ∈ d) }→B(a)
| 5. eq : EqDecider(A)
| 6. a : A
| 7. (a ∈ 1of(f) @ filter(λa.¬₂a ∈ dom(f);1of(g)))
| 8. f ∈ a:A fp-> B(a)
| 9. g ∈ a:A fp-> B(a)
| 10. a ∈ dom(f)
| ⊢  f(a) ∈ B(a)  by Thin -4
|
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. .....falsecase .....
. 1. A : Type
. 2. B : A→Type
. 3. f : d:A List×a:{a:A| (a ∈ d) }→B(a)
. 4. g : d:A List×a:{a:A| (a ∈ d) }→B(a)
. 5. eq : EqDecider(A)
. 6. a : A
. 7. (a ∈ 1of(f) @ filter(λa.¬₂a ∈ dom(f);1of(g)))
. 8. f ∈ a:A fp-> B(a)
. 9. g ∈ a:A fp-> B(a)
. 10. ¬a ∈ dom(f)
. ⊢  g(a) ∈ B(a)  by Assert a ∈ dom(g)
. |
. |\
. | .....assertion .....
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. | 1. A : Type
. | 2. B : A⟶Type
. | 3. f : d:A List×a:{a:A| (a ∈ d) }⟶B(a)
. | 4. g : d:A List×a:{a:A| (a ∈ d) }⟶B(a)
. | 5. eq : EqDecider(A)
. | 6. a : A
. | 7. (a ∈ 1of(f) @ filter(λa.¬₂a ∈ dom(f);1of(g)))
. | 8. f ∈ a:A fp-> B(a)
. | 9. g ∈ a:A fp-> B(a)
. | 10. ¬a ∈ dom(f)
. | ⊢  a ∈ dom(g)  by All (Unfold `fpf-dom`)
. | |
. | 1. A : Type
. | 2. B : A⟶Type
. | 3. f : d:A List×a:{a:A| (a ∈ d) }⟶B(a)
. | 4. g : d:A List×a:{a:A| (a ∈ d) }⟶B(a)
. | 5. eq : EqDecider(A)
. | 6. a : A
. | 7. (a ∈ 1of(f) @ filter(λa.¬₂deq-member(eq;a;1of(f));1of(g)))
. | 8. f ∈ a:A fp-> B(a)
. | 9. g ∈ a:A fp-> B(a)
. | 10. ¬deq-member(eq;a;1of(f))
. | ⊢  deq-member(eq;a;1of(g))
. | | by RWO
. | |    Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. | |    {member_append}
. | |    -4
. | |    THEN
. | |    Analyze -2
. | |
. | 1. A : Type
. | 2. B : A⟶Type
. | 3. f : d:A List×a:{a:A| (a ∈ d) }⟶B(a)
. | 4. g : d:A List×a:{a:A| (a ∈ d) }⟶B(a)
. | 5. eq : EqDecider(A)
. | 6. a : A
. | 7. (a ∈ 1of(f))
. | |  ∨ (a ∈ filter(λa.¬₂deq-member(eq;a;1of(f));1of(g)))
. | 8. f ∈ a:A fp-> B(a)
. | 9. g = g
. | 10. ¬deq-member(eq;a;1of(f))
. | ⊢  deq-member(eq;a;1of(g))
. | | by Thin -3
. | |    THEN
. | |    RWO
. | |    Thm* ∀eq:EqDecider(A), L:A List, x:A.
. | |         deq-member(eq;x;L) ⟺ (x ∈ L)
. | |    {assert_DASH_deq_DASH_member}
. | |    -1
. | |    THEN
. | |    Analyze -3
. | |
. | |\
. | | 1. A : Type
. | | 2. B : A⟶Type
```

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. | | 3. f : d:A List×a:{a:A| (a ∈ d) }→B(a)
. | | 4. g : d:A List×a:{a:A| (a ∈ d) }→B(a)
. | | 5. eq : EqDecider(A)
. | | 6. a : A
. | | 7. (a ∈ 1of(f))
. | | 8. g = g
. | | 9. ¬(a ∈ 1of(f))
. | | ⊢  deq-member(eq;a;1of(g))  by Auto
. | |
. | .\
. | . 1. A : Type
. | . 2. B : A→Type
. | . 3. f : d:A List×a:{a:A| (a ∈ d) }→B(a)
. | . 4. g : d:A List×a:{a:A| (a ∈ d) }→B(a)
. | . 5. eq : EqDecider(A)
. | . 6. a : A
. | . 7. (a ∈ filter(λa.¬₂deq-member(eq;a;1of(f));1of(g)))
. | . 8. g = g
. | . 9. ¬(a ∈ 1of(f))
. | . ⊢  deq-member(eq;a;1of(g))
. | . | by RWO
. | . |    Thm* ∀P:(T→𝔹), L:T List, x:T.
. | . |         (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
. | . |    {member_filter}
. | . |    -3
. | . |
. | . 1. A : Type
. | . 2. B : A→Type
. | . 3. f : d:A List×a:{a:A| (a ∈ d) }→B(a)
. | . 4. g : d:A List×a:{a:A| (a ∈ d) }→B(a)
. | . 5. eq : EqDecider(A)
. | . 6. a : A
. | . 7. (a ∈ 1of(g))
. | . 8. (λa.¬₂deq-member(eq;a;1of(f)))(a)
. | . 9. g = g
. | . 10. ¬(a ∈ 1of(f))
. | . ⊢  deq-member(eq;a;1of(g))
. | .   by RWO
. | .      Thm* ∀eq:EqDecider(A), L:A List, x:A.
. | .           deq-member(eq;x;L) ⟺ (x ∈ L)
. | .      {assert_DASH_deq_DASH_member}
. | .      0
. |
. .\
. . 1. A : Type
. . 2. B : A→Type
. . 3. f : d:A List×a:{a:A| (a ∈ d) }→B(a)
. . 4. g : d:A List×a:{a:A| (a ∈ d) }→B(a)
. . 5. eq : EqDecider(A)
. . 6. a : A
. . 7. (a ∈ 1of(f) @ filter(λa.¬₂a ∈ dom(f);1of(g)))
. . 8. f ∈ a:A fp-> B(a)
. . 9. g ∈ a:A fp-> B(a)
. . 10. ¬a ∈ dom(f)
. . 11. a ∈ dom(g)
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. . ⊢  g(a) ∈ B(a)  by Analyze -3 THEN Thin -3 THEN All (Fold ‘fpf‘)
END
```