```
decidable-exists-finite  THM

⊢  ∀T:Type, P:(T→Prop).
|  (∀x:T. Dec(P(x))) ⇒ finite-type(T) ⇒ Dec(∃x:T. P(x))  by Auto
|
.....decidable? .....
1. T : Type
2. P : T→Prop
3. ∀x:T. Dec(P(x))
4. finite-type(T)
⊢  Dec(∃x:T. P(x))
| by Analyze -1 THEN ExRepD
|    THEN
|    Assert ((∃x:T. P(x)) ⟺ (∃i:ℕn. P(f(i))))
|
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| .....assertion .....
| 1. T : Type
| 2. P : T→Prop
| 3. ∀x:T. Dec(P(x))
| 4. n : ℕ
| 5. f : ℕn→T
| 6. Surj(ℕn; T; f)
| ⊢  (∃x:T. P(x)) ⟺ (∃i:ℕn. P(f(i)))
| | by Unfold ‘surject‘ -1 THEN ExRepD
| |
| |\
| | 1. T : Type
| | 2. P : T→Prop
| | 3. ∀x:T. Dec(P(x))
| | 4. n : ℕ
| | 5. f : ℕn→T
| | 6. ∀b:T. ∃a:ℕn. f(a) = b
| | 7. x : T
| | 8. P(x)
| | ⊢  ∃i:ℕn. P(f(i))  by InstHyp [x] -3 THEN ParallelOp -1
| | |
| | 1. T : Type
| | 2. P : T→Prop
| | 3. ∀x:T. Dec(P(x))
| | 4. n : ℕ
| | 5. f : ℕn→T
| | 6. ∀b:T. ∃a:ℕn. f(a) = b
| | 7. x : T
| | 8. P(x)
| | 9. a : ℕn
| | 10. f(a) = x
| | ⊢  P(f(a))  by HypSubst -1 0
| |
| .\
| . 1. T : Type
| . 2. P : T→Prop
| . 3. ∀x:T. Dec(P(x))
| . 4. n : ℕ
| . 5. f : ℕn→T
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| . 6. ∀b:T. ∃a:ℕn. f(a) = b
| . 7. i : ℕn
| . 8. P(f(i))
| . ⊢  ∃x:T. P(x)  by InstConcl [f(i)]
|
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. 1. T : Type
. 2. P : T→Prop
. 3. ∀x:T. Dec(P(x))
. 4. n : ℕ
. 5. f : ℕn→T
. 6. Surj(ℕn; T; f)
. 7. (∃x:T. P(x)) ⟺ (∃i:ℕn. P(f(i)))
. ⊢  Dec(∃x:T. P(x))  by RW (SweepDnC (HypC -1)) 0
END
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