```
bi-graph-from_wf  THM

⊢  ∀G:(Id→𝔹), to,from:(|G|→(IdLnk List)), i:|G|.
|  bi-graph(G;to;from) ⇒ from(i) ∈ Edge(G) List
| by Auto THEN Unfold ‘bi-graph-from‘ 0
|
1. G : Id→𝔹
2. to : |G|→(IdLnk List)
3. from : |G|→(IdLnk List)
4. i : |G|
5. bi-graph(G;to;from)
⊢  from(i) ∈ Edge(G) List  by Assert (from ∈ |G|→(Edge(G) List))
|
|\
| .....assertion .....
| 1. G : Id→𝔹
| 2. to : |G|→(IdLnk List)
| 3. from : |G|→(IdLnk List)
| 4. |G|
| 5. bi-graph(G;to;from)
| ⊢  from ∈ |G|→(Edge(G) List)  by Thin -2
| |
| 1. G : Id→𝔹
| 2. to : |G|→(IdLnk List)
| 3. from : |G|→(IdLnk List)
| 4. bi-graph(G;to;from)
| ⊢  from ∈ |G|→(Edge(G) List)  by Ext2 THEN GenConclAtAddr [2]
| |
| 1. G : Id→𝔹
| 2. to : |G|→(IdLnk List)
| 3. from : |G|→(IdLnk List)
| 4. bi-graph(G;to;from)
| 5. x : |G|
| 6. v : IdLnk List
| 7. from(x) = v
| ⊢  v ∈ Edge(G) List  by Assert (∀l∈v.(l ∈ from(x)))
| |
| |\
| | .....assertion .....
| | 1. G : Id→𝔹
| | 2. to : |G|→(IdLnk List)
| | 3. from : |G|→(IdLnk List)
| | 4. bi-graph(G;to;from)
| | 5. x : |G|
| | 6. v : IdLnk List
| | 7. from(x) = v
| | ⊢  (∀l∈v.(l ∈ from(x)))  by Unfold ‘l_all‘ 0 THEN HypSubst -3 0
| |
| .\
| . 1. G : Id→𝔹
| . 2. to : |G|→(IdLnk List)
| . 3. from : |G|→(IdLnk List)
| . 4. bi-graph(G;to;from)
| . 5. x : |G|
| . 6. v : IdLnk List
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| . 7. from(x) = v
| . 8. (∀l∈v.(l ∈ from(x)))
| . ⊢  v ∈ Edge(G) List  by Thin -2 THEN ListInd -2 THEN Analyze 0
| . |
| . |\
| . | 1. G : Id→𝔹
| . | 2. to : |G|→(IdLnk List)
| . | 3. from : |G|→(IdLnk List)
| . | 4. bi-graph(G;to;from)
| . | 5. x : |G|
| . | 6. IdLnk List
| . | 7. (∀l∈nil.(l ∈ from(x)))
| . | ⊢  nil ∈ Edge(G) List  by Auto
| . |
| . .\
| . . 1. G : Id→𝔹
| . . 2. to : |G|→(IdLnk List)
| . . 3. from : |G|→(IdLnk List)
| . . 4. bi-graph(G;to;from)
| . . 5. x : |G|
| . . 6. IdLnk List
| . . 7. u : IdLnk
| . . 8. v1 : IdLnk List
| . . 9. (∀l∈v1.(l ∈ from(x))) ⇒ v1 ∈ Edge(G) List
| . . 10. (∀l∈[u / v1].(l ∈ from(x)))
| . . ⊢  [u / v1] ∈ Edge(G) List
| . . | by RWO
| . . |    Thm* ∀P:(T→Prop), x:T, L:T List.
| . . |          (∀y∈[x / L].P(y)) ⟺ P(x) & (∀y∈L.P(y))
| . . |    {l_all_cons}
| . . |    -1
| . . |    THEN
| . . |    Analyze -1
| . . |    THEN
| . . |    ThinTrivial
| . . |
| . . .....subterm .....
| . . 1. G : Id→𝔹
| . . 2. to : |G|→(IdLnk List)
| . . 3. from : |G|→(IdLnk List)
| . . 4. bi-graph(G;to;from)
| . . 5. x : |G|
| . . 6. IdLnk List
| . . 7. u : IdLnk
| . . 8. v1 : IdLnk List
| . . 9. (u ∈ from(x))
| . . 10. (∀l∈v1.(l ∈ from(x)))
| . . 11. v1 ∈ Edge(G) List
| . . ⊢  u ∈ Edge(G)  by Analyze
| . . |
| . . .....set predicate .....
| . . 1. G : Id→𝔹
| . . 2. to : |G|→(IdLnk List)
| . . 3. from : |G|→(IdLnk List)
| . . 4. bi-graph(G;to;from)
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| . . 5. x : |G|
| . . 6. IdLnk List
| . . 7. u : IdLnk
| . . 8. v1 : IdLnk List
| . . 9. (u ∈ from(x))
| . . 10. (∀l∈v1.(l ∈ from(x)))
| . . 11. v1 ∈ Edge(G) List
| . . ⊢  ∃i:|G|. (u ∈ from(i))  by InstConcl [x] THEN MaAuto
|
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. 1. G : Id→𝔹
. 2. to : |G|→(IdLnk List)
. 3. from : |G|→(IdLnk List)
. 4. i : |G|
. 5. bi-graph(G;to;from)
. 6. from ∈ |G|→(Edge(G) List)
. ⊢  from(i) ∈ Edge(G) List  by Auto
END
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