```
edge-to  THM

⊢  ∀G:(Id→𝔹), to,from:(|G|→(IdLnk List)), e:Edge(G), i:|G|.
|  bi-graph(G;to;from) ⇒ ((e ∈ to(i)) ⟺ destination(e) = i)
| by MaAuto
|
|\
| 1. G : Id→𝔹
| 2. to : |G|→(IdLnk List)
| 3. from : |G|→(IdLnk List)
| 4. e : Edge(G)
| 5. i : |G|
| 6. bi-graph(G;to;from)
| 7. (e ∈ to(i))
| ⊢  destination(e) = i
| | by DupHyp -2 THEN Unfold ‘bi-graph‘ -1 THEN InstHyp [i] -1
| |    THEN
| |    Unfold ‘l_all‘ -2
| |    THEN
| |    InstHyp [e] -2
| |    THEN
| |    MaAuto
| |
| .....antecedent .....
| 1. G : Id→𝔹
| 2. to : |G|→(IdLnk List)
| 3. from : |G|→(IdLnk List)
| 4. e : Edge(G)
| 5. i : |G|
| 6. bi-graph(G;to;from)
| 7. (e ∈ to(i))
| 8. ∀i:|G|.
| |  (∀l∈to(i).destination(l) = i
| |  & G(source(l))
| |  & (l ∈ from(source(l)))
| |  & (lnk-inv(l) ∈ from(i)))
| |  & (∀l∈from(i).source(l) = i
| |    & G(destination(l))
| |    & (l ∈ to(destination(l)))
| |    & (lnk-inv(l) ∈ to(i)))
| 9. ∀l:IdLnk.
| |  (l ∈ to(i))
| |  ⇒
| |  destination(l) = i
| |  & G(source(l))
| |  & (l ∈ from(source(l)))
| |  & (lnk-inv(l) ∈ from(i))
| 10. (∀l∈from(i).source(l) = i
| |   & G(destination(l))
| |   & (l ∈ to(destination(l)))
| |   & (lnk-inv(l) ∈ to(i)))
| ⊢  (e ∈ to(i))  by Assert (e ∈ to(i))
| |
| |\
| | .....assertion .....
```

```
| | 1. G : Id→𝔹
| | 2. to : |G|→(IdLnk List)
| | 3. from : |G|→(IdLnk List)
| | 4. e : Edge(G)
| | 5. i : |G|
| | 6. bi-graph(G;to;from)
| | 7. (e ∈ to(i))
| | 8. ∀i:|G|.
| |    (∀l∈to(i).destination(l) = i
| |    & G(source(l))
| |    & (l ∈ from(source(l)))
| |    & (lnk-inv(l) ∈ from(i)))
| |    & (∀l∈from(i).source(l) = i
| |      & G(destination(l))
| |      & (l ∈ to(destination(l)))
| |      & (lnk-inv(l) ∈ to(i)))
| | 9. ∀l:IdLnk.
| |    (l ∈ to(i))
| |    ⇒
| |    destination(l) = i
| |    & G(source(l))
| |    & (l ∈ from(source(l)))
| |    & (lnk-inv(l) ∈ from(i))
| | 10. (∀l∈from(i).source(l) = i
| |     & G(destination(l))
| |     & (l ∈ to(destination(l)))
| |     & (lnk-inv(l) ∈ to(i)))
| | ⊢  (e ∈ to(i))
| |   by Using [‘A’,Edge(G)]
| |      (BackThru
| |       Thm* ∀L:A List, x:A. (A ⊆r B) ⇒ (x ∈ L) ⇒ (x ∈ L)
| |       {l_member_subtype})
| |      THEN
| |      MaAuto
| |      THEN
| |      Analyze 0
| |      THEN
| |      MaAuto
| |
| .\
| . 1. G : Id→𝔹
| . 2. to : |G|→(IdLnk List)
| . 3. from : |G|→(IdLnk List)
| . 4. e : Edge(G)
| . 5. i : |G|
| . 6. bi-graph(G;to;from)
| . 7. (e ∈ to(i))
| . 8. ∀i:|G|.
| .    (∀l∈to(i).destination(l) = i
| .    & G(source(l))
| .    & (l ∈ from(source(l)))
| .    & (lnk-inv(l) ∈ from(i)))
| .    & (∀l∈from(i).source(l) = i
| .      & G(destination(l))
| .      & (l ∈ to(destination(l)))
```

```
| .      & (lnk-inv(l) ∈ to(i)))
| . 9. ∀l:IdLnk.
| .    (l ∈ to(i))
| .    ⇒
| .    destination(l) = i
| .    & G(source(l))
| .    & (l ∈ from(source(l)))
| .    & (lnk-inv(l) ∈ from(i))
| . 10. (∀l∈from(i).source(l) = i
| .     & G(destination(l))
| .     & (l ∈ to(destination(l)))
| .     & (lnk-inv(l) ∈ to(i)))
| . 11. (e ∈ to(i))
| . ⊢  (e ∈ to(i))  by Unfold ‘bi-graph-to‘ -1
|
.\
. 1. G : Id→𝔹
. 2. to : |G|→(IdLnk List)
. 3. from : |G|→(IdLnk List)
. 4. e : Edge(G)
. 5. i : |G|
. 6. bi-graph(G;to;from)
. 7. destination(e) = i
. ⊢  (e ∈ to(i))  by RevHypSubst’ -1 0 THEN Thin -1 THEN Thin -2
. |
. 1. G : Id→𝔹
. 2. to : |G|→(IdLnk List)
. 3. from : |G|→(IdLnk List)
. 4. e : Edge(G)
. 5. bi-graph(G;to;from)
. ⊢  (e ∈ to(destination(e)))
. | by Assert Dec((e ∈ to(destination(e))))
. |
. |\
. | .....assertion .....
. | 1. G : Id→𝔹
. | 2. to : |G|→(IdLnk List)
. | 3. from : |G|→(IdLnk List)
. | 4. e : Edge(G)
. | 5. bi-graph(G;to;from)
. | ⊢  Dec((e ∈ to(destination(e))))  by ProveDecidable1 THENA MaAuto
. | |
. | |\
. | | .....decidable? .....
. | | 1. G : Id→𝔹
. | | 2. to : |G|→(IdLnk List)
. | | 3. from : |G|→(IdLnk List)
. | | 4. Edge(G)
. | | 5. bi-graph(G;to;from)
. | | 6. x : Edge(G)
. | | 7. y : Edge(G)
. | | ⊢  Dec(x = y)
. | |    by AssertBY Dec(x = y) MaAuto THEN RepeatFor 4 ParallelLast
. | |       THEN
. | |       Fold ‘member‘ 0
```

```
. | |      THEN
. | |      MaAuto
. | |
. | .\
. | . .....wf .....
. | . 1. G : Id→𝔹
. | . 2. to : |G|→(IdLnk List)
. | . 3. from : |G|→(IdLnk List)
. | . 4. e : Edge(G)
. | . 5. bi-graph(G;to;from)
. | . ⊢  destination(e) ∈ |G|
. | .   by BackThru
. | .      Thm* ∀T:(Id→𝔹), to,from:(|T|→(IdLnk List)), u:Edge(T).
. | .           bi-graph(T;to;from) ⇒ destination(u) ∈ |T|
. | .      {dst_DASH_edge}
. |
. .\
. . 1. G : Id→𝔹
. . 2. to : |G|→(IdLnk List)
. . 3. from : |G|→(IdLnk List)
. . 4. e : Edge(G)
. . 5. bi-graph(G;to;from)
. . 6. Dec((e ∈ to(destination(e))))
. . ⊢  (e ∈ to(destination(e)))
. . | by Analyze -1 THEN Try Trivial THEN Assert False
. . |
. . .....assertion .....
. . 1. G : Id→𝔹
. . 2. to : |G|→(IdLnk List)
. . 3. from : |G|→(IdLnk List)
. . 4. e : Edge(G)
. . 5. bi-graph(G;to;from)
. . 6. ¬(e ∈ to(destination(e)))
. . ⊢  False
. . | by AssertBY (e ∈ Edge(G)) Auto THEN DVar ‘e’ THEN ExRepD
. . |    THEN
. . |    Analyze -2
. . |    THEN
. . |    DupHyp -2
. . |    THEN
. . |    Unfold ‘bi-graph‘ -1
. . |    THEN
. . |    InstHyp [i] -1
. . |    THEN
. . |    Unfold ‘l_all‘ -1
. . |    THEN
. . |    InstHyp [e] -1
. . |
. . 1. G : Id→𝔹
. . 2. to : |G|→(IdLnk List)
. . 3. from : |G|→(IdLnk List)
. . 4. e : IdLnk
. . 5. i : |G|
. . 6. (e ∈ from(i))
. . 7. bi-graph(G;to;from)
```

```
. . 8. e ∈ Edge(G)
. . 9. ∀i:|G|.
. . |  (∀l∈to(i).destination(l) = i
. . |  & G(source(l))
. . |  & (l ∈ from(source(l)))
. . |  & (lnk-inv(l) ∈ from(i)))
. . |  & (∀l∈from(i).source(l) = i
. . |    & G(destination(l))
. . |    & (l ∈ to(destination(l)))
. . |    & (lnk-inv(l) ∈ to(i)))
. . 10. (∀l∈to(i).destination(l) = i
. . |   & G(source(l))
. . |   & (l ∈ from(source(l)))
. . |   & (lnk-inv(l) ∈ from(i)))
. . 11. ∀l:IdLnk.
. . |   (l ∈ from(i))
. . |   ⇒
. . |   source(l) = i
. . |   & G(destination(l))
. . |   & (l ∈ to(destination(l)))
. . |   & (lnk-inv(l) ∈ to(i))
. . 12. source(e) = i
. . 13. G(destination(e))
. . 14. (e ∈ to(destination(e)))
. . 15. (lnk-inv(e) ∈ to(i))
. . ⊢  (e ∈ to(destination(e)))
. . | by Thin -1
. . |    THEN
. . |    RepeatFor 3
. . |    (ParallelLast
. . |     THEN
. . |     Try
. . |     (BackThru
. . |      Thm* ∀T:(Id→𝔹), to,from:(|T|→(IdLnk List)), u:Edge(T).
. . |           bi-graph(T;to;from) ⇒ destination(u) ∈ |T|
. . |      {dst_DASH_edge}))
. . |    THEN
. . |    Try (Unfold ‘bi-graph-to‘ 0)
. . |    THEN
. . |    Try Trivial
. . |
. . 1. G : Id→𝔹
. . 2. to : |G|→(IdLnk List)
. . 3. from : |G|→(IdLnk List)
. . 4. e : IdLnk
. . 5. i : |G|
. . 6. (e ∈ from(i))
. . 7. bi-graph(G;to;from)
. . 8. e ∈ Edge(G)
. . 9. ∀i:|G|.
. .    (∀l∈to(i).destination(l) = i
. .    & G(source(l))
. .    & (l ∈ from(source(l)))
. .    & (lnk-inv(l) ∈ from(i)))
. .    & (∀l∈from(i).source(l) = i
```

```
. .      & G(destination(l))
. .      & (l ∈ to(destination(l)))
. .      & (lnk-inv(l) ∈ to(i)))
. . 10. (∀l∈to(i).destination(l) = i
. .     & G(source(l))
. .     & (l ∈ from(source(l)))
. .     & (lnk-inv(l) ∈ from(i)))
. . 11. ∀l:IdLnk.
. .     (l ∈ from(i))
. .     ⇒
. .     source(l) = i
. .     & G(destination(l))
. .     & (l ∈ to(destination(l)))
. .     & (lnk-inv(l) ∈ to(i))
. . 12. source(e) = i
. . 13. G(destination(e))
. . 14. i1 : ℕ
. . 15. e = (to(destination(e)))[i1]
. . ⊢  e = (to(destination(e)))[i1]  by RevHypSubst' -1 0
END
```