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rdist-property  THM

⊢  ∀R:(Id→𝔹), in,out:(|R|→IdLnk), i,j:|R|.
|  ring(R;in;out)
|  ⇒
|  λx.n(x)^d(i;j)(i) = j & (∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j)
| by UnivCD
|
1. R : Id→𝔹
2. in : |R|→IdLnk
3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ring(R;in;out)
⊢  λx.n(x)^d(i;j)(i) = j & (∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j)
| by AssertBY ((λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹)
|    (Reduce 0 THEN Try (DoSubsume THEN Analyze 0 THEN Analyze -1))
|
1. R : Id→𝔹
2. in : |R|→IdLnk
3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ring(R;in;out)
7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
⊢  λx.n(x)^d(i;j)(i) = j & (∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j)
| by Assert (∃n:ℕ. (λk.λx.n(x)^k+1(i) = j)(n))
|
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| .....assertion .....
| 1. R : Id→𝔹
| 2. in : |R|→IdLnk
| 3. out : |R|→IdLnk
| 4. i : |R|
| 5. j : |R|
| 6. ring(R;in;out)
| 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
| ⊢  ∃n:ℕ. (λk.λx.n(x)^k+1(i) = j)(n)  by Reduce 0
| |
| 1. R : Id→𝔹
| 2. in : |R|→IdLnk
| 3. out : |R|→IdLnk
| 4. i : |R|
| 5. j : |R|
| 6. ring(R;in;out)
| 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
| ⊢  ∃n:ℕ. λx.n(x)^n+1(i) = j
|   by DupHyp -2 THEN Unfold `ring` -1 THEN ExRepD
|      THEN
|      InstHyp [i;j] -2
|      THEN
|      ExRepD
|      THEN
|      InstConcl [k-1]
|      THEN
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|      Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
|      THEN
|      Subst ((k-1+1) ∼ k) 0
|      THENL
|      [Auto;Id]
|      THEN
|      RW assert_pushdownC 0
|      THEN
|      Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
|      THEN
|      Unfold `rnext` 0
|      THEN
|      Trivial
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. 1. R : Id→𝔹
. 2. in : |R|→IdLnk
. 3. out : |R|→IdLnk
. 4. i : |R|
. 5. j : |R|
. 6. ring(R;in;out)
. 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. 8. ∃n:ℕ. (λk.λx.n(x)^k+1(i) = j)(n)
. ⊢  λx.n(x)^d(i;j)(i) = j & (∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j)
. | by Inst
. |    Thm* ∀f:(ℕ→𝔹).
. |         (∃n:ℕ. f(n)) ⇒ f(mu(f)) & (∀i:ℕ. i<mu(f) ⇒ ¬f(i))
. |    {mu_DASH_property}
. |    [λk.λx.n(x)^k+1(i) = j]
. |    THENA
. |    Trivial
. |    THEN
. |    Inst Thm* ∀f:(ℕ→𝔹). (∃n:ℕ. f(n)) ⇒ mu(f) ∈ ℕ    {mu_wf}
. |    [λk.λx.n(x)^k+1(i) = j]
. |    THENA
. |    Trivial
. |
. 1. R : Id→𝔹
. 2. in : |R|→IdLnk
. 3. out : |R|→IdLnk
. 4. i : |R|
. 5. j : |R|
. 6. ring(R;in;out)
. 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. 8. ∃n:ℕ. (λk.λx.n(x)^k+1(i) = j)(n)
. 9. (λk.λx.n(x)^k+1(i) = j)(mu(λk.λx.n(x)^k+1(i) = j))
. |  & (∀i@0:ℕ.
. |     i@0<mu(λk.λx.n(x)^k+1(i) = j)
. |     ⇒
. |     ¬(λk.λx.n(x)^k+1(i) = j)(i@0))
. 10. mu(λk.λx.n(x)^k+1(i) = j) ∈ ℕ
. ⊢  λx.n(x)^d(i;j)(i) = j & (∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j)
. | by All Reduce
. |    THEN
. |    Subst (mu(λk.λx.n(x)^k+1(i) = j) ∼ (d(i;j)-1)) -2
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. | .....equality .....
. | 1. R : Id→𝔹
. | 2. in : |R|→IdLnk
. | 3. out : |R|→IdLnk
. | 4. i : |R|
. | 5. j : |R|
. | 6. ring(R;in;out)
. | 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. | 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. | 9. λx.n(x)^mu(λk.λx.n(x)^k+1(i) = j)+1(i) = j
. |    & (∀i@0:ℕ.
. |       i@0<mu(λk.λx.n(x)^k+1(i) = j) ⇒ ¬λx.n(x)^i@0+1(i) = j)
. | 10. mu(λk.λx.n(x)^k+1(i) = j) ∈ ℕ
. | ⊢  mu(λk.λx.n(x)^k+1(i) = j) ∼ (d(i;j)-1)
. |   by Unfold ‘rdist‘ 0 THEN GenConclAtAddr [1]
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. . 1. R : Id→𝔹
. . 2. in : |R|→IdLnk
. . 3. out : |R|→IdLnk
. . 4. i : |R|
. . 5. j : |R|
. . 6. ring(R;in;out)
. . 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. . 9. λx.n(x)^d(i;j)-1+1(i) = j
. . |  & (∀i@0:ℕ. i@0<d(i;j)-1 ⇒ ¬λx.n(x)^i@0+1(i) = j)
. . 10. mu(λk.λx.n(x)^k+1(i) = j) ∈ ℕ
. . ⊢  λx.n(x)^d(i;j)(i) = j & (∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j)
. . | by Thin -1 THEN Parallel0p -1
. . |
. . |\
. . | 1. R : Id→𝔹
. . | 2. in : |R|→IdLnk
. . | 3. out : |R|→IdLnk
. . | 4. i : |R|
. . | 5. j : |R|
. . | 6. ring(R;in;out)
. . | 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . | 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. . | 9. λx.n(x)^d(i;j)-1+1(i) = j
. . | ⊢  λx.n(x)^d(i;j)(i) = j  by Subst ((d(i;j)-1+1) ∼ d(i;j)) -1
. . | |
. . | |\
. . | | .....equality .....
. . | | 1. R : Id→𝔹
. . | | 2. in : |R|→IdLnk
. . | | 3. out : |R|→IdLnk
. . | | 4. i : |R|
. . | | 5. j : |R|
. . | | 6. ring(R;in;out)
. . | | 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . | | 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
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. . | | 9. λx.n(x)^d(i;j)-1+1(i) = j
. . | | ⊢  (d(i;j)-1+1) ∼ d(i;j)  by Auto
. . | |
. . | .\
. . | . 1. R : Id→𝔹
. . | . 2. in : |R|→IdLnk
. . | . 3. out : |R|→IdLnk
. . | . 4. i : |R|
. . | . 5. j : |R|
. . | . 6. ring(R;in;out)
. . | . 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . | . 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. . | . 9. λx.n(x)^d(i;j)(i) = j
. . | . ⊢  λx.n(x)^d(i;j)(i) = j
. . | . | by RW assert_pushdownC -1
. . | . |    THEN
. . | . |    Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
. . | . |
. . | . 1. R : Id→𝔹
. . | . 2. in : |R|→IdLnk
. . | . 3. out : |R|→IdLnk
. . | . 4. i : |R|
. . | . 5. j : |R|
. . | . 6. ring(R;in;out)
. . | . 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . | . 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. . | . 9. λx.n(x)^d(i;j)(i) = j
. . | . ⊢  λx.n(x)^d(i;j)(i) = j
. . | .   by StrongHypSubst -1 0 THEN Unfold `rset` 0 THEN DVar `j`
. . | .      THEN
. . | .      Analyze
. . | .      THEN
. . | .      HypSubst -1 0
. . |
. . .\
. . . 1. R : Id→𝔹
. . . 2. in : |R|→IdLnk
. . . 3. out : |R|→IdLnk
. . . 4. i : |R|
. . . 5. j : |R|
. . . 6. ring(R;in;out)
. . . 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . . 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. . . 9. ∀i@0:ℕ. i@0<d(i;j)-1 ⇒ ¬λx.n(x)^i@0+1(i) = j
. . . ⊢  ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | by Auto THEN InstHyp [k-1] -3 THEN Subst ((k-1+1) ∼ k) -1
. . . |    THENL
. . . |    [Auto;Id]
. . . |
. . . 1. R : Id→𝔹
. . . 2. in : |R|→IdLnk
. . . 3. out : |R|→IdLnk
. . . 4. i : |R|
. . . 5. j : |R|
. . . 6. ring(R;in;out)
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. . . 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . . 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. . . 9. ∀i@0:ℕ. i@0<d(i;j)-1 ⇒ ¬λx.n(x)^i@0+1(i) = j
. . . 10. k : ℕ+
. . . 11. k<d(i;j)
. . . 12. ¬λx.n(x)^k(i) = j
. . . ⊢  ¬λx.n(x)^k(i) = j
. . . | by RW assert_pushdownC -1
. . . |    THEN
. . . |    Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
. . . |
. . . 1. R : Id→𝔹
. . . 2. in : |R|→IdLnk
. . . 3. out : |R|→IdLnk
. . . 4. i : |R|
. . . 5. j : |R|
. . . 6. ring(R;in;out)
. . . 7. (λk.λx.n(x)^k+1(i) = j) ∈ ℕ→𝔹
. . . 8. ∃n:ℕ. λx.n(x)^n+1(i) = j
. . . 9. ∀i@0:ℕ. i@0<d(i;j)-1 ⇒ ¬λx.n(x)^i@0+1(i) = j
. . . 10. k : ℕ+
. . . 11. k<d(i;j)
. . . 12. ¬λx.n(x)^k(i) = j
. . . ⊢  ¬λx.n(x)^k(i) = j
. . .   by ParallelOp -1 THEN HypSubst -1 0 THEN Try (Fold `member` 0)
. . .      THEN
. . .      Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
END
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