```
rdist-rprev  THM

⊢  ∀R:(Id→𝔹), in,out:(|R|→IdLnk), i,j:|R|.
|  ring(R;in;out) ⇒ ¬i = p(j) ⇒ d(i;p(j)) = d(i;j)-1  by Auto
|
1. R : Id→𝔹
2. in : |R|→IdLnk
3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ring(R;in;out)
7. ¬i = p(j)
⊢  d(i;p(j)) = d(i;j)-1
| by Inst Thm: rdist_DASH_property [R;in;out;i;j]
|    THEN
|    Inst Thm: rdist_DASH_property [R;in;out;i;p(j)]
|
1. R : Id→𝔹
2. in : |R|→IdLnk
3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ring(R;in;out)
7. ¬i = p(j)
8. λx.n(x)^d(i;j)(i) = j
9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
10. λx.n(x)^d(i;p(j))(i) = p(j)
11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
⊢  d(i;p(j)) = d(i;j)-1  by Assert (n(p(j)) = j)
|
|\
| .....assertion .....
| 1. R : Id→𝔹
| 2. in : |R|→IdLnk
| 3. out : |R|→IdLnk
| 4. i : |R|
| 5. j : |R|
| 6. ring(R;in;out)
| 7. ¬i = p(j)
| 8. λx.n(x)^d(i;j)(i) = j
| 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
| 10. λx.n(x)^d(i;p(j))(i) = p(j)
| 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
| ⊢  n(p(j)) = j
|   by BackThru
|      Thm* ∀R:(Id→𝔹), in,out:(|R|→IdLnk), j:|R|.
|           ring(R;in;out) ⇒ n(p(j)) = j
|      {rnext_DASH_rprev}
|
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. 1. R : Id→𝔹
. 2. in : |R|→IdLnk
. 3. out : |R|→IdLnk
. 4. i : |R|
. 5. j : |R|
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. 6. ring(R;in;out)
. 7. ¬i = p(j)
. 8. λx.n(x)^d(i;j)(i) = j
. 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. 10. λx.n(x)^d(i;p(j))(i) = p(j)
. 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. 12. n(p(j)) = j
. ⊢  d(i;p(j)) = d(i;j)-1  by Assert (λx.n(x)^d(i;p(j))+1(i) = j)
. |
. |\
. | .....assertion .....
. | 1. R : Id→𝔹
. | 2. in : |R|→IdLnk
. | 3. out : |R|→IdLnk
. | 4. i : |R|
. | 5. j : |R|
. | 6. ring(R;in;out)
. | 7. ¬i = p(j)
. | 8. λx.n(x)^d(i;j)(i) = j
. | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. | 12. n(p(j)) = j
. | ⊢  λx.n(x)^d(i;p(j))+1(i) = j
. | | by Subst ((d(i;p(j))+1) ~ (1+d(i;p(j)))) 0 THENL [Auto;Id]
. | |    THEN
. | |    Inst
. | |    Thm* ∀n,m:ℕ, f:(T→T). f^n+m = f^n o f^m    {fun_exp_add}
. | |    [|R|;1;d(i;p(j));λx.n(x)]
. | |    THEN
. | |    HypSubst -1 0
. | |    THEN
. | |    Reduce 0
. | |
. | 1. R : Id→𝔹
. | 2. in : |R|→IdLnk
. | 3. out : |R|→IdLnk
. | 4. i : |R|
. | 5. j : |R|
. | 6. ring(R;in;out)
. | 7. ¬i = p(j)
. | 8. λx.n(x)^d(i;j)(i) = j
. | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. | 12. n(p(j)) = j
. | 13. λx.n(x)^1+d(i;p(j)) = λx.n(x)^1 o λx.n(x)^d(i;p(j))
. | ⊢  n(λx.n(x)^d(i;p(j))(i)) = j  by HypSubst 10 0
. |
. .\
. . 1. R : Id→𝔹
. . 2. in : |R|→IdLnk
. . 3. out : |R|→IdLnk
. . 4. i : |R|
. . 5. j : |R|
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. . 6. ring(R;in;out)
. . 7. ¬i = p(j)
. . 8. λx.n(x)^d(i;j)(i) = j
. . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . 12. n(p(j)) = j
. . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . ⊢  d(i;p(j)) = d(i;j)-1  by Assert (d(i;j)≤d(i;p(j))+1)
. . |
. . |\
. . | .....assertion .....
. . | 1. R : Id→𝔹
. . | 2. in : |R|→IdLnk
. . | 3. out : |R|→IdLnk
. . | 4. i : |R|
. . | 5. j : |R|
. . | 6. ring(R;in;out)
. . | 7. ¬i = p(j)
. . | 8. λx.n(x)^d(i;j)(i) = j
. . | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . | 12. n(p(j)) = j
. . | 13. λx.n(x)^d(i;p(j))+1(i) = j
. . | ⊢  d(i;j)≤d(i;p(j))+1  by SupposeNot
. . | |
. . | 1. R : Id→𝔹
. . | 2. in : |R|→IdLnk
. . | 3. out : |R|→IdLnk
. . | 4. i : |R|
. . | 5. j : |R|
. . | 6. ring(R;in;out)
. . | 7. ¬i = p(j)
. . | 8. λx.n(x)^d(i;j)(i) = j
. . | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . | 12. n(p(j)) = j
. . | 13. λx.n(x)^d(i;p(j))+1(i) = j
. . | 14. ¬d(i;j)≤d(i;p(j))+1
. . | ⊢  d(i;j)≤d(i;p(j))+1  by InstHyp [d(i;p(j))+1] 9
. . |
. . .\
. . . 1. R : Id→𝔹
. . . 2. in : |R|→IdLnk
. . . 3. out : |R|→IdLnk
. . . 4. i : |R|
. . . 5. j : |R|
. . . 6. ring(R;in;out)
. . . 7. ¬i = p(j)
. . . 8. λx.n(x)^d(i;j)(i) = j
. . . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
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. . . 12. n(p(j)) = j
. . . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . 14. d(i;j)≤d(i;p(j))+1
. . . ⊢  d(i;p(j)) = d(i;j)-1  by Assert (d(i;p(j))≤d(i;j)-1)
. . . |
. . . |\
. . . | .....assertion .....
. . . | 1. R : Id→𝔹
. . . | 2. in : |R|→IdLnk
. . . | 3. out : |R|→IdLnk
. . . | 4. i : |R|
. . . | 5. j : |R|
. . . | 6. ring(R;in;out)
. . . | 7. ¬i = p(j)
. . . | 8. λx.n(x)^d(i;j)(i) = j
. . . | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | 12. n(p(j)) = j
. . . | 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | 14. d(i;j)≤d(i;p(j))+1
. . . | ⊢  d(i;p(j))≤d(i;j)-1  by SupposeNot
. . . | |
. . . | 1. R : Id→𝔹
. . . | 2. in : |R|→IdLnk
. . . | 3. out : |R|→IdLnk
. . . | 4. i : |R|
. . . | 5. j : |R|
. . . | 6. ring(R;in;out)
. . . | 7. ¬i = p(j)
. . . | 8. λx.n(x)^d(i;j)(i) = j
. . . | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | 12. n(p(j)) = j
. . . | 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | 14. d(i;j)≤d(i;p(j))+1
. . . | 15. ¬d(i;p(j))≤d(i;j)-1
. . . | ⊢  d(i;p(j))≤d(i;j)-1  by Assert (d(i;j)-1 ∈ ℕ+)
. . . | |
. . . | |\
. . . | | .....assertion .....
. . . | | 1. R : Id→𝔹
. . . | | 2. in : |R|→IdLnk
. . . | | 3. out : |R|→IdLnk
. . . | | 4. i : |R|
. . . | | 5. j : |R|
. . . | | 6. ring(R;in;out)
. . . | | 7. ¬i = p(j)
. . . | | 8. λx.n(x)^d(i;j)(i) = j
. . . | | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | | 12. n(p(j)) = j
. . . | | 13. λx.n(x)^d(i;p(j))+1(i) = j
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. . . | | 14. d(i;j)≤d(i;p(j))+1
. . . | | 15. ¬d(i;p(j))≤d(i;j)-1
. . . | | ⊢  d(i;j)-1 ∈ ℕ+  by Decide (d(i;j) = 1)
. . . | | |
. . . | | 1. R : Id→𝔹
. . . | | 2. in : |R|→IdLnk
. . . | | 3. out : |R|→IdLnk
. . . | | 4. i : |R|
. . . | | 5. j : |R|
. . . | | 6. ring(R;in;out)
. . . | | 7. ¬i = p(j)
. . . | | 8. λx.n(x)^d(i;j)(i) = j
. . . | | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | | 12. n(p(j)) = j
. . . | | 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | | 14. d(i;j)≤d(i;p(j))+1
. . . | | 15. ¬d(i;p(j))≤d(i;j)-1
. . . | | 16. d(i;j) = 1
. . . | | ⊢  d(i;j)-1 ∈ ℕ+  by StrongHypSubst -1 8 THEN Reduce 8
. . . | | |
. . . | | 1. R : Id→𝔹
. . . | | 2. in : |R|→IdLnk
. . . | | 3. out : |R|→IdLnk
. . . | | 4. i : |R|
. . . | | 5. j : |R|
. . . | | 6. ring(R;in;out)
. . . | | 7. ¬i = p(j)
. . . | | 8. n(i) = j
. . . | | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | | 12. n(p(j)) = j
. . . | | 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | | 14. d(i;j)≤d(i;p(j))+1
. . . | | 15. ¬d(i;p(j))≤d(i;j)-1
. . . | | 16. d(i;j) = 1
. . . | | ⊢  0<d(i;j)-1  by Assert (∀i,j:|R|. n(i) = n(j) ⇒ i = j)
. . . | | |
. . . | | |\
. . . | | | .....assertion .....
. . . | | | 1. R : Id→𝔹
. . . | | | 2. in : |R|→IdLnk
. . . | | | 3. out : |R|→IdLnk
. . . | | | 4. i : |R|
. . . | | | 5. j : |R|
. . . | | | 6. ring(R;in;out)
. . . | | | 7. ¬i = p(j)
. . . | | | 8. n(i) = j
. . . | | | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | | | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | | | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | | | 12. n(p(j)) = j
. . . | | | 13. λx.n(x)^d(i;p(j))+1(i) = j
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. . . | | | 14. d(i;j)≤d(i;p(j))+1
. . . | | | 15. ¬d(i;p(j))≤d(i;j)-1
. . . | | | 16. d(i;j) = 1
. . . | | | ⊢  ∀i,j:|R|. n(i) = n(j) ⇒ i = j
. . . | | |   by Inst
. . . | | |      Thm* ∀R:(Id→𝔹), in,out:(|R|→IdLnk).
. . . | | |           ring(R;in;out)
. . . | | |           ⇒
. . . | | |           (∀i,j:|R|. n(i) = n(j) ⇒ i = j)
. . . | | |      {rnext_DASH_one_DASH_one}
. . . | | |      [R;in;out]
. . . | | |      THEN
. . . | | |      Try Trivial
. . . | | |
. . . | | .\
. . . | | . 1. R : Id→𝔹
. . . | | . 2. in : |R|→IdLnk
. . . | | . 3. out : |R|→IdLnk
. . . | | . 4. i : |R|
. . . | | . 5. j : |R|
. . . | | . 6. ring(R;in;out)
. . . | | . 7. ¬i = p(j)
. . . | | . 8. n(i) = j
. . . | | . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | | . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | | . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | | . 12. n(p(j)) = j
. . . | | . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | | . 14. d(i;j)≤d(i;p(j))+1
. . . | | . 15. ¬d(i;p(j))≤d(i;j)-1
. . . | | . 16. d(i;j) = 1
. . . | | . 17. ∀i,j:|R|. n(i) = n(j) ⇒ i = j
. . . | | . ⊢  0<d(i;j)-1  by Analyze 7 THEN BackThru -1
. . . | |
. . . | .\
. . . | . 1. R : Id→𝔹
. . . | . 2. in : |R|→IdLnk
. . . | . 3. out : |R|→IdLnk
. . . | . 4. i : |R|
. . . | . 5. j : |R|
. . . | . 6. ring(R;in;out)
. . . | . 7. ¬i = p(j)
. . . | . 8. λx.n(x)^d(i;j)(i) = j
. . . | . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | . 12. n(p(j)) = j
. . . | . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | . 14. d(i;j)≤d(i;p(j))+1
. . . | . 15. ¬d(i;p(j))≤d(i;j)-1
. . . | . 16. d(i;j)-1 ∈ ℕ+
. . . | . ⊢  d(i;p(j))≤d(i;j)-1
. . . | . | by InstHyp [d(i;j)-1] 11 THEN Analyze -1
. . . | . |
. . . | . 1. R : Id→𝔹
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. . . | . 2. in : |R|→IdLnk
. . . | . 3. out : |R|→IdLnk
. . . | . 4. i : |R|
. . . | . 5. j : |R|
. . . | . 6. ring(R;in;out)
. . . | . 7. ¬i = p(j)
. . . | . 8. λx.n(x)^d(i;j)(i) = j
. . . | . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | . 12. n(p(j)) = j
. . . | . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | . 14. d(i;j)≤d(i;p(j))+1
. . . | . 15. ¬d(i;p(j))≤d(i;j)-1
. . . | . 16. d(i;j)-1 ∈ ℕ+
. . . | . ⊢  λx.n(x)^d(i;j)-1(i) = p(j)
. . . | . | by Inst
. . . | . |    Thm* ∀n,m:ℕ, f:(T→T). f^n+m = f^n o f^m
. . . | . |    {fun_exp_add}
. . . | . |    [|R|;1;d(i;j)-1;λx.n(x)]
. . . | . |    THEN
. . . | . |    Reduce -1
. . . | . |    THEN
. . . | . |    Subst ((1+d(i;j)-1) ∼ d(i;j)) -1
. . . | . |    THENL
. . . | . |    [Auto;Id]
. . . | . |
. . . | . 1. R : Id→𝔹
. . . | . 2. in : |R|→IdLnk
. . . | . 3. out : |R|→IdLnk
. . . | . 4. i : |R|
. . . | . 5. j : |R|
. . . | . 6. ring(R;in;out)
. . . | . 7. ¬i = p(j)
. . . | . 8. λx.n(x)^d(i;j)(i) = j
. . . | . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | . 12. n(p(j)) = j
. . . | . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | . 14. d(i;j)≤d(i;p(j))+1
. . . | . 15. ¬d(i;p(j))≤d(i;j)-1
. . . | . 16. d(i;j)-1 ∈ ℕ+
. . . | . 17. λx.n(x)^d(i;j) = (λx.n(x)) o (λx.x) o λx.n(x)^d(i;j)-1
. . . | . ⊢  λx.n(x)^d(i;j)-1(i) = p(j)
. . . | . | by HypSubst -1 8 THEN Reduce 8
. . . | . |
. . . | . 1. R : Id→𝔹
. . . | . 2. in : |R|→IdLnk
. . . | . 3. out : |R|→IdLnk
. . . | . 4. i : |R|
. . . | . 5. j : |R|
. . . | . 6. ring(R;in;out)
. . . | . 7. ¬i = p(j)
. . . | . 8. n(λx.n(x)^d(i;j)-1(i)) = j
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. . . | . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | . 12. n(p(j)) = j
. . . | . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | . 14. d(i;j)≤d(i;p(j))+1
. . . | . 15. ¬d(i;p(j))≤d(i;j)-1
. . . | . 16. d(i;j)-1 ∈ ℕ+
. . . | . 17. λx.n(x)^d(i;j) = (λx.n(x)) o (λx.x) o λx.n(x)^d(i;j)-1
. . . | . ⊢  λx.n(x)^d(i;j)-1(i) = p(j)
. . . | . | by Assert (∀i,j:|R|. n(i) = n(j) ⇒ i = j)
. . . | . |
. . . | . |\
. . . | . | .....assertion .....
. . . | . | 1. R : Id→𝔹
. . . | . | 2. in : |R|→IdLnk
. . . | . | 3. out : |R|→IdLnk
. . . | . | 4. i : |R|
. . . | . | 5. j : |R|
. . . | . | 6. ring(R;in;out)
. . . | . | 7. ¬i = p(j)
. . . | . | 8. n(λx.n(x)^d(i;j)-1(i)) = j
. . . | . | 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | . | 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | . | 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | . | 12. n(p(j)) = j
. . . | . | 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | . | 14. d(i;j)≤d(i;p(j))+1
. . . | . | 15. ¬d(i;p(j))≤d(i;j)-1
. . . | . | 16. d(i;j)-1 ∈ ℕ+
. . . | . | 17. λx.n(x)^d(i;j) = (λx.n(x)) o (λx.x) o λx.n(x)^d(i;j)-1
. . . | . | ⊢  ∀i,j:|R|. n(i) = n(j) ⇒ i = j
. . . | . |   by Inst
. . . | . |      Thm* ∀R:(Id→𝔹), in,out:(|R|→IdLnk).
. . . | . |           ring(R;in;out)
. . . | . |           ⇒
. . . | . |           (∀i,j:|R|. n(i) = n(j) ⇒ i = j)
. . . | . |      {rnext_DASH_one_DASH_one}
. . . | . |      [R;in;out]
. . . | . |      THEN
. . . | . |      Try Trivial
. . . | . |
. . . | . .\
. . . | . . 1. R : Id→𝔹
. . . | . . 2. in : |R|→IdLnk
. . . | . . 3. out : |R|→IdLnk
. . . | . . 4. i : |R|
. . . | . . 5. j : |R|
. . . | . . 6. ring(R;in;out)
. . . | . . 7. ¬i = p(j)
. . . | . . 8. n(λx.n(x)^d(i;j)-1(i)) = j
. . . | . . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . | . . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . | . . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . | . . 12. n(p(j)) = j
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. . . | . . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . | . . 14. d(i;j)≤d(i;p(j))+1
. . . | . . 15. ¬d(i;p(j))≤d(i;j)-1
. . . | . . 16. d(i;j)-1 ∈ ℕ+
. . . | . . 17. λx.n(x)^d(i;j) = (λx.n(x)) o (λx.x) o λx.n(x)^d(i;j)-1
. . . | . . 18. ∀i,j:|R|. n(i) = n(j) ⇒ i = j
. . . | . . ⊢  λx.n(x)^d(i;j)-1(i) = p(j)  by BackThru -1
. . . |
. . . .\
. . . . 1. R : Id→𝔹
. . . . 2. in : |R|→IdLnk
. . . . 3. out : |R|→IdLnk
. . . . 4. i : |R|
. . . . 5. j : |R|
. . . . 6. ring(R;in;out)
. . . . 7. ¬i = p(j)
. . . . 8. λx.n(x)^d(i;j)(i) = j
. . . . 9. ∀k:ℕ+. k<d(i;j) ⇒ ¬λx.n(x)^k(i) = j
. . . . 10. λx.n(x)^d(i;p(j))(i) = p(j)
. . . . 11. ∀k:ℕ+. k<d(i;p(j)) ⇒ ¬λx.n(x)^k(i) = p(j)
. . . . 12. n(p(j)) = j
. . . . 13. λx.n(x)^d(i;p(j))+1(i) = j
. . . . 14. d(i;j)≤d(i;p(j))+1
. . . . 15. d(i;p(j))≤d(i;j)-1
. . . . ⊢  d(i;p(j)) = d(i;j)-1  by Auto
END
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