```
rdist_wf  THM

⊢  ∀R:(Id→𝔹), in,out:(|R|→IdLnk), i,j:|R|.
|  ring(R;in;out) ⇒ d(i;j) ∈ ℕ+  by UnivCD
|
1. R : Id→𝔹
2. in : |R|→IdLnk
3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ring(R;in;out)
⊢  d(i;j) ∈ ℕ+
| by Unfold `rdist` 0 THEN GenConcl ((λx.n(x)) = n)
|    THEN
|    GenConcl (mu(λk.n^k+1(i) = j) = z)
|    THEN
|    Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
|
1. R : Id→𝔹
2. in : |R|→IdLnk
3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ring(R;in;out)
7. n : |R|→|R|
8. (λx.n(x)) = n
⊢  ∃n1:ℕ. (λk.n^k+1(i) = j)(n1)
| by Reduce 0 THEN Unfold `ring` -3 THEN ExRepD THEN InstHyp [i;j] -4
|    THEN
|    ExRepD
|
1. R : Id→𝔹
2. in : |R|→IdLnk
3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ∀i:|R|.
|  R(source(in(i))) & R(destination(out(i)))
|  & source(out(i)) = i
|    & destination(in(i)) = i
|    & in(destination(out(i))) = out(i)
|    & out(source(in(i))) = in(i)
7. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
8. |R|
9. n : |R|→|R|
10. (λx.n(x)) = n
11. k : ℕ+
12. λx.destination(out(x))^k(i) = j
⊢  ∃n1:ℕ. n^n1+1(i) = j
| by InstConcl [k-1]
|    THEN
|    Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
|
1. R : Id→𝔹
2. in : |R|→IdLnk
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3. out : |R|→IdLnk
4. i : |R|
5. j : |R|
6. ∀i:|R|.
|  R(source(in(i))) & R(destination(out(i)))
|  & source(out(i)) = i
|    & destination(in(i)) = i
|    & in(destination(out(i))) = out(i)
|    & out(source(in(i))) = in(i)
7. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
8. |R|
9. n : |R|→|R|
10. (λx.n(x)) = n
11. k : ℕ+
12. λx.destination(out(x))^k(i) = j
⊢  n^k-1+1(i) = j  by Subst ((k-1+1) ∼ k) 0
|
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| .....equality .....
| 1. R : Id→𝔹
| 2. in : |R|→IdLnk
| 3. out : |R|→IdLnk
| 4. i : |R|
| 5. j : |R|
| 6. ∀i:|R|.
|    R(source(in(i))) & R(destination(out(i)))
|    & source(out(i)) = i
|      & destination(in(i)) = i
|      & in(destination(out(i))) = out(i)
|      & out(source(in(i))) = in(i)
| 7. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
| 8. |R|
| 9. n : |R|→|R|
| 10. (λx.n(x)) = n
| 11. k : ℕ+
| 12. λx.destination(out(x))^k(i) = j
| ⊢  (k-1+1) ∼ k  by Auto
|
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. 1. R : Id→𝔹
. 2. in : |R|→IdLnk
. 3. out : |R|→IdLnk
. 4. i : |R|
. 5. j : |R|
. 6. ∀i:|R|.
. |  R(source(in(i))) & R(destination(out(i)))
. |  & source(out(i)) = i
. |    & destination(in(i)) = i
. |    & in(destination(out(i))) = out(i)
. |    & out(source(in(i))) = in(i)
. 7. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
. 8. |R|
. 9. n : |R|→|R|
. 10. (λx.n(x)) = n
. 11. k : ℕ+
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. 12. λx.destination(out(x))^k(i) = j
. ⊢  n^k(i) = j
. | by RevHypSubst -3 0
. |    THEN
. |    Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
. |
. 1. R : Id→𝔹
. 2. in : |R|→IdLnk
. 3. out : |R|→IdLnk
. 4. i : |R|
. 5. j : |R|
. 6. ∀i:|R|.
.    R(source(in(i))) & R(destination(out(i)))
.    & source(out(i)) = i
.      & destination(in(i)) = i
.      & in(destination(out(i))) = out(i)
.      & out(source(in(i))) = in(i)
. 7. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
. 8. |R|
. 9. n : |R|→|R|
. 10. (λx.n(x)) = n
. 11. k : ℕ+
. 12. λx.destination(out(x))^k(i) = j
. ⊢  λx.n(x)^k(i) = j
.   by RW assert_pushdownC 0
.      THEN
.      Try (DoSubsume THEN Analyze 0 THEN Analyze -1)
.      THEN
.      Unfold ‘rnext‘ 0
.      THEN
.      Trivial
END
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