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rnext-rprev  THM

⊢  ∀R:(Id→𝔹), in,out:(|R|→IdLnk), j:|R|.
|  ring(R;in;out) ⇒ n(p(j)) = j
| by Auto THEN Unfold ‘ring‘ -1 THEN Unfolds [‘rnext‘;‘rprev‘] 0
|    THEN
|    ExRepD
|    THEN
|    InstHyp [j] 5
|    THEN
|    ExRepD
|    THEN
|    StrongHypSubst -1 0
|
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| 1. R : Id→𝔹
| 2. in : |R|→IdLnk
| 3. out : |R|→IdLnk
| 4. j : |R|
| 5. ∀i:|R|.
|    R(source(in(i))) & R(destination(out(i)))
|    & source(out(i)) = i
|      & destination(in(i)) = i
|      & in(destination(out(i))) = out(i)
|      & out(source(in(i))) = in(i)
| 6. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
| 7. |R|
| 8. R(source(in(j)))
| 9. R(destination(out(j)))
| 10. source(out(j)) = j
| 11. destination(in(j)) = j
| 12. in(destination(out(j))) = out(j)
| 13. out(source(in(j))) = in(j)
| ⊢  destination(in(j)) = j
|   by StrongHypSubst -3 0 THEN DVar ‘j’ THEN Unfold ‘rset‘ 0
|      THEN
|      Analyze
|      THEN
|      HypSubst -1 0
|
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. .....subterm .....
. 1. R : Id→𝔹
. 2. in : |R|→IdLnk
. 3. out : |R|→IdLnk
. 4. j : |R|
. 5. ∀i:|R|.
. |  R(source(in(i))) & R(destination(out(i)))
. |  & source(out(i)) = i
. |    & destination(in(i)) = i
. |    & in(destination(out(i))) = out(i)
. |    & out(source(in(i))) = in(i)
. 6. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
. 7. |R|
. 8. R(source(in(j)))
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. 9. R(destination(out(j)))
. 10. source(out(j)) = j
. 11. destination(in(j)) = j
. 12. in(destination(out(j))) = out(j)
. 13. out(source(in(j))) = in(j)
. 14. z : IdLnk
. 15. z = in(j)
. ⊢  destination(z) ∈ |R|
. | by DVar ‘j’ THEN Unfold ‘rset‘ 0 THEN Analyze THEN HypSubst -1 0
. |
. 1. R : Id→𝔹
. 2. in : |R|→IdLnk
. 3. out : |R|→IdLnk
. 4. j : Id
. 5. R(j)
. 6. ∀i:|R|.
.    R(source(in(i))) & R(destination(out(i)))
.    & source(out(i)) = i
.      & destination(in(i)) = i
.      & in(destination(out(i))) = out(i)
.      & out(source(in(i))) = in(i)
. 7. ∀i,j:|R|. ∃k:ℕ+. λx.destination(out(x))^k(i) = j
. 8. |R|
. 9. R(source(in(j)))
. 10. R(destination(out(j)))
. 11. source(out(j)) = j
. 12. destination(in(j)) = j
. 13. in(destination(out(j))) = out(j)
. 14. out(source(in(j))) = in(j)
. 15. z : IdLnk
. 16. z = in(j)
. ⊢  R(destination(in(j)))  by HypSubst -5 0
END
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