```
s-unconditional-send1-rule  THM

⊢  ∀x,tg:Id, k:Knd, l:IdLnk, T,A,B:Type.
|  (rcv(l; tg) = k ⇒ T = B)
|  ⇒
|  (∀f:(A→B→T).
|   @source(l): ma-single-sends1(A;
|                                 B;
|                                 T;
|                                 x;
|                                 k;
|                                 l;
|                                 tg;
|                                 (λa,b. [(f(a,b))]))
|   ∈ Dsys
|   & (∀D:Dsys.
|      @source(l): ma-single-sends1(A;
|                                   B;
|                                   T;
|                                   x;
|                                   k;
|                                   l;
|                                   tg;
|                                   (λa,b. [(f(a,b))])) ⊆ D
|      ⇒
|      D
|      realizes es.(vartype(source(l);x) ⊆r A)
|                 & (∀e:E.
|                    loc(e) = source(l)
|                    ⇒
|                    kind(e) = k ⇒ (valtype(e) ⊆r B))
|                 & (∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T))
|                 & (∀e:E.
|                    loc(e) = source(l)
|                    ⇒
|                    kind(e) = k
|                    ⇒
|                    (∃e':E.
|                     kind(e') = rcv(l; tg)
|                     & sender(e') = e
|                       & (∀e'':E.
|                          kind(e'') = rcv(l; tg)
|                          ⇒
|                          sender(e'') = e ⇒ e'' = e')
|                       & val(e') = f((x when e),val(e))))))
| by Inst Thm: s_DASH_sends_DASH_rule1 []
|    THEN
|    RepeatFor 7 (Parallel0p -1 THEN Thin -3)
|    THEN
|    Analyze 0
|    THEN
|    Analyze 0
|    THEN
|    InstHyp [λa,b. [(f(a,b))]] -3
|    THEN
```

```
|    Thin -4
|    THEN
|    Analyze -1
|    THEN
|    Analyze 0
|    THEN
|    Try Trivial
|    THEN
|    ParallelOp -2
|    THEN
|    Thin -4
|    THEN
|    ParallelOp -1
|    THEN
|    ParallelOp -1
|    THEN
|    ParallelOp -1
|    THEN
|    Thin -3
|    THEN
|    ParallelOp -1
|    THEN
|    RepeatFor 2 (ParallelOp -1 THEN Thin -3)
|    THEN
|    ParallelOp -1
|    THEN
|    ExRepD
|    THEN
|    BetterSplitAndConcl
|    THEN
|    Try Trivial
|    THEN
|    All Reduce
|
1. x : Id
2. tg : Id
3. k : Knd
4. l : IdLnk
5. T : Type
6. A : Type
7. B : Type
8. rcv(l; tg) = k ⇒ T = B
9. f : A⟶B⟶T
10. @source(l): ma-single-sends1(A;
|                                B;
|                                T;
|                                x;
|                                k;
|                                l;
|                                tg;
|                                (λa,b. [(f(a,b))]))
|   ∈ Dsys
11. @source(l): ma-single-sends1(A;
|                                B;
|                                T;
```

```
|                                x;
|                                k;
|                                l;
|                                tg;
|                                (λa,b. [(f(a,b))]))
|   ∈ Dsys
12. D : Dsys
13. @source(l): ma-single-sends1(A;
|                                B;
|                                T;
|                                x;
|                                k;
|                                l;
|                                tg;
|                                (λa,b. [(f(a,b))])) ⊆ D
14. D' : Dsys
15. D ⊆ D'
16. w : World
17. p : FairFifo
18. PossibleWorld(D';w)
19. vartype(source(l);x) ⊆r A
20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
22. ∀e:E.
|   loc(e) = source(l)
|   ⇒
|   kind(e) = k
|   ⇒
|   (∃L:{e':E| kind(e') = rcv(l; tg) } List.
|    (∀e':E. (e' ∈ L) ⟺ kind(e') = rcv(l; tg) & sender(e') = e)
|    & (∀e1,e2:E. e1 before e2 ∈ L ⇒ (e1 <loc e2))
|    & map(λe'.val(e');L) = [(f((x when e),val(e)))])
23. (vartype(source(l);x) ⊆r A)
|   & (∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B))
|   & (∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T))
⊢  ∀e:E.
|  loc(e) = source(l)
|  ⇒
|  kind(e) = k
|  ⇒
|  (∃e':E.
|   kind(e') = rcv(l; tg)
|   & sender(e') = e
|     & (∀e'':E.
|        kind(e'') = rcv(l; tg) ⇒ sender(e'') = e ⇒ e'' = e')
|     & val(e') = f((x when e),val(e)))
| by ParallelOp -2 THEN Thin -4 THEN ParallelLast THEN ParallelLast
|
1. x : Id
2. tg : Id
3. k : Knd
4. l : IdLnk
5. T : Type
6. A : Type
7. B : Type
```

```
8. rcv(l; tg) = k ⇒ T = B
9. f : A→B→T
10. @source(l): ma-single-sends1(A;
|                                B;
|                                T;
|                                x;
|                                k;
|                                l;
|                                tg;
|                                (λa,b. [(f(a,b))]))
|   ∈ Dsys
11. @source(l): ma-single-sends1(A;
|                                B;
|                                T;
|                                x;
|                                k;
|                                l;
|                                tg;
|                                (λa,b. [(f(a,b))]))
|   ∈ Dsys
12. D : Dsys
13. @source(l): ma-single-sends1(A;
|                                B;
|                                T;
|                                x;
|                                k;
|                                l;
|                                tg;
|                                (λa,b. [(f(a,b))]) ⊆ D
14. D' : Dsys
15. D ⊆ D'
16. w : World
17. p : FairFifo
18. PossibleWorld(D';w)
19. vartype(source(l);x) ⊆r A
20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
22. (vartype(source(l);x) ⊆r A)
|   & (∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B))
|   & (∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T))
23. e : E
24. loc(e) = source(l)
25. kind(e) = k
26. ∃L:{e':E| kind(e') = rcv(l; tg) } List.
|   (∀e':E. (e' ∈ L) ⟺ kind(e') = rcv(l; tg) & sender(e') = e)
|   & (∀e1,e2:E. e1 before e2 ∈ L ⇒ (e1 <loc e2))
|   & map(λe'.val(e');L) = [(f((x when e),val(e)))]
⊢  ∃e':E.
|  kind(e') = rcv(l; tg)
|  & sender(e') = e
|    & (∀e'':E. kind(e'') = rcv(l; tg) ⇒ sender(e'') = e ⇒ e'' = e')
|    & val(e') = f((x when e),val(e))
| by ExRepD THEN Assert (||L|| = 1)
|
|\
```

```
| .....assertion .....
| 1. x : Id
| 2. tg : Id
| 3. k : Knd
| 4. l : IdLnk
| 5. T : Type
| 6. A : Type
| 7. B : Type
| 8. rcv(l; tg) = k ⇒ T = B
| 9. f : A→B→T
| 10. @source(l): ma-single-sends1(A;
|                                  B;
|                                  T;
|                                  x;
|                                  k;
|                                  l;
|                                  tg;
|                                  (λa,b. [(f(a,b))]))
|     ∈ Dsys
| 11. @source(l): ma-single-sends1(A;
|                                  B;
|                                  T;
|                                  x;
|                                  k;
|                                  l;
|                                  tg;
|                                  (λa,b. [(f(a,b))]))
|     ∈ Dsys
| 12. D : Dsys
| 13. @source(l): ma-single-sends1(A;
|                                  B;
|                                  T;
|                                  x;
|                                  k;
|                                  l;
|                                  tg;
|                                  (λa,b. [(f(a,b))])) ⊆ D
| 14. D' : Dsys
| 15. D ⊆ D'
| 16. w : World
| 17. p : FairFifo
| 18. PossibleWorld(D';w)
| 19. vartype(source(l);x) ⊆r A
| 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
| 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
| 22. vartype(source(l);x) ⊆r A
| 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
| 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
| 25. e : E
| 26. loc(e) = source(l)
| 27. kind(e) = k
| 28. L : {e':E| kind(e') = rcv(l; tg) } List
| 29. ∀e':E. (e' ∈ L) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
| 30. ∀e1,e2:E. e1 before e2 ∈ L ⇒ (e1 <loc e2)
| 31. map(λe'.val(e');L) = [(f((x when e),val(e)))]
```

```
| ⊢  ||L|| = 1
|   by ApFunToHypEquands ‘Z’ ||Z|| ℤ -1 THEN Reduce -1
|      THEN
|      RWO
|      Thm* ∀f:Top, T:Type, L:T List. ||map(f;L)|| ∼ ||L||
|      {length_DASH_map}
|      -1
|
.\
. 1. x : Id
. 2. tg : Id
. 3. k : Knd
. 4. l : IdLnk
. 5. T : Type
. 6. A : Type
. 7. B : Type
. 8. rcv(l; tg) = k ⇒ T = B
. 9. f : A→B→T
. 10. @source(l): ma-single-sends1(A;
. |                                B;
. |                                T;
. |                                x;
. |                                k;
. |                                l;
. |                                tg;
. |                                (λa,b. [(f(a,b))]))
. |   ∈ Dsys
. 11. @source(l): ma-single-sends1(A;
. |                                B;
. |                                T;
. |                                x;
. |                                k;
. |                                l;
. |                                tg;
. |                                (λa,b. [(f(a,b))]))
. |   ∈ Dsys
. 12. D : Dsys
. 13. @source(l): ma-single-sends1(A;
. |                                B;
. |                                T;
. |                                x;
. |                                k;
. |                                l;
. |                                tg;
. |                                (λa,b. [(f(a,b))])) ⊆ D
. 14. D’ : Dsys
. 15. D ⊆ D’
. 16. w : World
. 17. p : FairFifo
. 18. PossibleWorld(D’;w)
. 19. vartype(source(l);x) ⊆r A
. 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. 22. vartype(source(l);x) ⊆r A
. 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
```

```
. 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. 25. e : E
. 26. loc(e) = source(l)
. 27. kind(e) = k
. 28. L : {e':E| kind(e') = rcv(l; tg) } List
. 29. ∀e':E. (e' ∈ L) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. 30. ∀e1,e2:E. e1 before e2 ∈ L ⇒ (e1 <loc e2)
. 31. map(λe'.val(e');L) = [(f((x when e),val(e)))]
. 32. ||L|| = 1
. ⊢  ∃e':E.
. |  kind(e') = rcv(l; tg)
. |  & sender(e') = e
. |    & (∀e'':E.
. |       kind(e'') = rcv(l; tg) ⇒ sender(e'') = e ⇒ e'' = e')
. |    & val(e') = f((x when e),val(e))
. | by DVar `L' THEN Reduce -1 THEN DVar `v' THEN Reduce -1
. |    THEN
. |    Thin -1
. |    THEN
. |    Reduce -1
. |    THEN
. |    Thin -2
. |
. 1. x : Id
. 2. tg : Id
. 3. k : Knd
. 4. l : IdLnk
. 5. T : Type
. 6. A : Type
. 7. B : Type
. 8. rcv(l; tg) = k ⇒ T = B
. 9. f : A→B→T
. 10. @source(l): ma-single-sends1(A;
. |                                B;
. |                                T;
. |                                x;
. |                                k;
. |                                l;
. |                                tg;
. |                                (λa,b. [(f(a,b))]))
. |   ∈ Dsys
. 11. @source(l): ma-single-sends1(A;
. |                                B;
. |                                T;
. |                                x;
. |                                k;
. |                                l;
. |                                tg;
. |                                (λa,b. [(f(a,b))]))
. |   ∈ Dsys
. 12. D : Dsys
. 13. @source(l): ma-single-sends1(A;
. |                                B;
. |                                T;
. |                                x;
```

```
. |                                k;
. |                                l;
. |                                tg;
. |                                (λa,b. [(f(a,b))]))) ⊆ D
. 14. D' : Dsys
. 15. D ⊆ D'
. 16. w : World
. 17. p : FairFifo
. 18. PossibleWorld(D';w)
. 19. vartype(source(l);x) ⊆r A
. 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. 22. vartype(source(l);x) ⊆r A
. 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. 25. e : E
. 26. loc(e) = source(l)
. 27. kind(e) = k
. 28. u : {e':E| kind(e') = rcv(l; tg) }
. 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. 30. [val(u)] = [(f((x when e),val(e)))]
. ⊢  ∃e':E.
. |  kind(e') = rcv(l; tg)
. |  & sender(e') = e
. |    & (∀e'':E.
. |       kind(e'') = rcv(l; tg) ⇒ sender(e'') = e ⇒ e'' = e')
. |    & val(e') = f((x when e),val(e))  by InstConcl [u]
. |
. |\
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
```

```
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                   B;
. |                                   T;
. |                                   x;
. |                                   k;
. |                                   l;
. |                                   tg;
. |                                   (λa,b. [(f(a,b))])) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | ⊢  kind(u) = rcv(l; tg)  by DVar `u'
. |
. |\
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                   B;
. |                                   T;
. |                                   x;
. |                                   k;
. |                                   l;
. |                                   tg;
. |                                   (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                   B;
. |                                   T;
. |                                   x;
. |                                   k;
. |                                   l;
. |                                   tg;
```

```
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))])) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | ⊢  sender(u) = e
. |   by InstHyp [u] -2 THEN Analyze -2 THEN ExRepD
. |      THEN
. |      RW0 Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a    {member_singleton} 0
. |
. |\
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
```

```
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))])) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | 31. e'' : E
. | 32. kind(e'') = rcv(l; tg)
. | 33. sender(e'') = e
. | ⊢  e'' = u  by RWO "member_singleton<" 0 THEN BackThruSomeHyp
. |
. |\
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
```

```
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                   B;
. |                                   T;
. |                                   x;
. |                                   k;
. |                                   l;
. |                                   tg;
. |                                   (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                   B;
. |                                   T;
. |                                   x;
. |                                   k;
. |                                   l;
. |                                   tg;
. |                                   (λa,b. [(f(a,b))])) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | 31. e'' : E
. | 32. kind(e'') = rcv(l; tg)
. | ⊢ isrcv(e'')
. |   by (FwdThru
. |       Thm* ∀es:ES, l:IdLnk, tg:Id, e:E.
. |            kind(e) = rcv(l; tg)
. |            ⇒
. |            isrcv(e)
. |            & lnk(e) = l & tag(e) = tg & loc(sender(e)) = source(l)
. |       {es_DASH_kind_DASH_rcv}
. |       [-1])
. |      THEN
. |      ExRepD
. |
. |\
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
```

```
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. | |                                B;
. | |                                T;
. | |                                x;
. | |                                k;
. | |                                l;
. | |                                tg;
. | |                                (λa,b. [(f(a,b))]))
. | |   ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. | |                                B;
. | |                                T;
. | |                                x;
. | |                                k;
. | |                                l;
. | |                                tg;
. | |                                (λa,b. [(f(a,b))]))
. | |   ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. | |                                B;
. | |                                T;
. | |                                x;
. | |                                k;
. | |                                l;
. | |                                tg;
. | |                                (λa,b. [(f(a,b))]) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | ⊢  val(u) = f((x when e),val(e))
. | | by Subst (val(u) ∼ hd([val(u)])) 0
. | |
. | |\
. | | .....equality .....
. | | 1. x : Id
```

```
. | | 2. tg : Id
. | | 3. k : Knd
. | | 4. l : IdLnk
. | | 5. T : Type
. | | 6. A : Type
. | | 7. B : Type
. | | 8. rcv(l; tg) = k ⇒ T = B
. | | 9. f : A→B→T
. | | 10. @source(l): ma-single-sends1(A;
. | |                                  B;
. | |                                  T;
. | |                                  x;
. | |                                  k;
. | |                                  l;
. | |                                  tg;
. | |                                  (λa,b. [(f(a,b))]))
. | |     ∈ Dsys
. | | 11. @source(l): ma-single-sends1(A;
. | |                                  B;
. | |                                  T;
. | |                                  x;
. | |                                  k;
. | |                                  l;
. | |                                  tg;
. | |                                  (λa,b. [(f(a,b))]))
. | |     ∈ Dsys
. | | 12. D : Dsys
. | | 13. @source(l): ma-single-sends1(A;
. | |                                  B;
. | |                                  T;
. | |                                  x;
. | |                                  k;
. | |                                  l;
. | |                                  tg;
. | |                                  (λa,b. [(f(a,b))])) ⊆ D
. | | 14. D' : Dsys
. | | 15. D ⊆ D'
. | | 16. w : World
. | | 17. p : FairFifo
. | | 18. PossibleWorld(D';w)
. | | 19. vartype(source(l);x) ⊆r A
. | | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | | 22. vartype(source(l);x) ⊆r A
. | | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | | 25. e : E
. | | 26. loc(e) = source(l)
. | | 27. kind(e) = k
. | | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | | 30. [val(u)] = [(f((x when e),val(e)))]
. | | ⊢  val(u) ∼ hd([val(u)])  by Reduce 0
. | |
. | .\
```

```
. | . 1. x : Id
. | . 2. tg : Id
. | . 3. k : Knd
. | . 4. l : IdLnk
. | . 5. T : Type
. | . 6. A : Type
. | . 7. B : Type
. | . 8. rcv(l; tg) = k ⇒ T = B
. | . 9. f : A→B→T
. | . 10. @source(l): ma-single-sends1(A;
. | .                                  B;
. | .                                  T;
. | .                                  x;
. | .                                  k;
. | .                                  l;
. | .                                  tg;
. | .                                  (λa,b. [(f(a,b))]))
. | .     ∈ Dsys
. | . 11. @source(l): ma-single-sends1(A;
. | .                                  B;
. | .                                  T;
. | .                                  x;
. | .                                  k;
. | .                                  l;
. | .                                  tg;
. | .                                  (λa,b. [(f(a,b))]))
. | .     ∈ Dsys
. | . 12. D : Dsys
. | . 13. @source(l): ma-single-sends1(A;
. | .                                  B;
. | .                                  T;
. | .                                  x;
. | .                                  k;
. | .                                  l;
. | .                                  tg;
. | .                                  (λa,b. [(f(a,b))])) ⊆ D
. | . 14. D' : Dsys
. | . 15. D ⊆ D'
. | . 16. w : World
. | . 17. p : FairFifo
. | . 18. PossibleWorld(D';w)
. | . 19. vartype(source(l);x) ⊆r A
. | . 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | . 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | . 22. vartype(source(l);x) ⊆r A
. | . 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | . 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | . 25. e : E
. | . 26. loc(e) = source(l)
. | . 27. kind(e) = k
. | . 28. u : {e':E| kind(e') = rcv(l; tg) }
. | . 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | . 30. [val(u)] = [(f((x when e),val(e)))]
. | . ⊢ hd([val(u)]) = f((x when e),val(e))
. | .   by (StrongHypSubst -1 0 THEN Reduce 0 THEN MaAuto
```

```
. | .       THEN
. | .       HypSubst -1 0)
. | .      THEN
. | .      (Reduce 0)
. |
. |\
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))])) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
```

```
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | 31. e' : E
. | 32. kind(e') = rcv(l; tg)
. | ⊢  isrcv(e')
. |   by (FwdThru
. |       Thm* ∀es:ES, l:IdLnk, tg:Id, e:E.
. |            kind(e) = rcv(l; tg)
. |            ⇒
. |            isrcv(e)
. |            & lnk(e) = l & tag(e) = tg & loc(sender(e)) = source(l)
. |       {es_DASH_kind_DASH_rcv}
. |       [-1])
. |      THEN
. |      ExRepD
. |
. |\
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))])) ⊆ D
```

```
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | 31. e' : E
. | 32. kind(e') = rcv(l; tg)
. | 33. e'' : E
. | 34. kind(e'') = rcv(l; tg)
. | ⊢  isrcv(e'')
. |   by (FwdThru
. |       Thm* ∀es:ES, l:IdLnk, tg:Id, e:E.
. |            kind(e) = rcv(l; tg)
. |            ⇒
. |            isrcv(e)
. |            & lnk(e) = l & tag(e) = tg & loc(sender(e)) = source(l)
. |       {es_DASH_kind_DASH_rcv}
. |       [-1])
. |      THEN
. |      ExRepD
. |
. |\
. | .....subterm .....
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                  B;
```

```
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))])) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | 31. e' : E
. | 32. kind(e') = rcv(l; tg)
. | ⊢  val(e') ∈ T  by MaAuto
. |
. |\
. | .....subterm .....
. | 1. x : Id
. | 2. tg : Id
. | 3. k : Knd
. | 4. l : IdLnk
. | 5. T : Type
. | 6. A : Type
. | 7. B : Type
. | 8. rcv(l; tg) = k ⇒ T = B
. | 9. f : A→B→T
. | 10. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
```

```
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 11. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))]))
. |     ∈ Dsys
. | 12. D : Dsys
. | 13. @source(l): ma-single-sends1(A;
. |                                  B;
. |                                  T;
. |                                  x;
. |                                  k;
. |                                  l;
. |                                  tg;
. |                                  (λa,b. [(f(a,b))])) ⊆ D
. | 14. D' : Dsys
. | 15. D ⊆ D'
. | 16. w : World
. | 17. p : FairFifo
. | 18. PossibleWorld(D';w)
. | 19. vartype(source(l);x) ⊆r A
. | 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 22. vartype(source(l);x) ⊆r A
. | 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. | 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. | 25. e : E
. | 26. loc(e) = source(l)
. | 27. kind(e) = k
. | 28. u : {e':E| kind(e') = rcv(l; tg) }
. | 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. | 30. [val(u)] = [(f((x when e),val(e)))]
. | 31. e' : E
. | 32. kind(e') = rcv(l; tg)
. | ⊢  (x when e) ∈ A  by MaAuto
. |
. .\
. . .....subterm .....
. . 1. x : Id
. . 2. tg : Id
. . 3. k : Knd
. . 4. l : IdLnk
. . 5. T : Type
. . 6. A : Type
. . 7. B : Type
. . 8. rcv(l; tg) = k ⇒ T = B
. . 9. f : A→B→T
. . 10. @source(l): ma-single-sends1(A;
. .                                  B;
. .                                  T;
```

```
. .                                  x;
. .                                  k;

. .                                  l;
. .                                  tg;
. .                                  (λa,b. [(f(a,b))]))
. .     ∈ Dsys
. . 11. @source(l): ma-single-sends1(A;
. .                                  B;
. .                                  T;
. .                                  x;
. .                                  k;
. .                                  l;
. .                                  tg;
. .                                  (λa,b. [(f(a,b))]))
. .     ∈ Dsys
. . 12. D : Dsys
. . 13. @source(l): ma-single-sends1(A;
. .                                  B;
. .                                  T;
. .                                  x;
. .                                  k;
. .                                  l;
. .                                  tg;
. .                                  (λa,b. [(f(a,b))])) ⊆ D
. . 14. D' : Dsys
. . 15. D ⊆ D'
. . 16. w : World
. . 17. p : FairFifo
. . 18. PossibleWorld(D';w)
. . 19. vartype(source(l);x) ⊆r A
. . 20. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. . 21. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. . 22. vartype(source(l);x) ⊆r A
. . 23. ∀e:E. loc(e) = source(l) ⇒ kind(e) = k ⇒ (valtype(e) ⊆r B)
. . 24. ∀e:E. kind(e) = rcv(l; tg) ⇒ (valtype(e) ⊆r T)
. . 25. e : E
. . 26. loc(e) = source(l)
. . 27. kind(e) = k
. . 28. u : {e':E| kind(e') = rcv(l; tg) }
. . 29. ∀e':E. (e' ∈ [u]) ⟺ kind(e') = rcv(l; tg) & sender(e') = e
. . 30. [val(u)] = [(f((x when e),val(e)))]
. . 31. e' : E
. . 32. kind(e') = rcv(l; tg)
. . ⊢  val(e) ∈ B  by MaAuto
END
```