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causal_fusion  THM

⊢  ∀E:TaggedEventStruct. PTrue fuses Causal(E)
| by ...
THEN Auto
|
1. E : TaggedEventStruct
2. tr : |E| List
3. ∀m:Label. Causal(E)(<tr>_m)
4. True
⊢  Causal(E)(tr)
| by RWO "P_causal_iff" 0 THENA Auto THEN Unfold ‘l_all‘ 0 THEN Auto

|    THEN AllHyps (InstHyp [tag(E)(x)]) THENA Auto

|    THEN RWO "P_causal_iff" (-1) THENA Auto
|
1. E : TaggedEventStruct
2. tr : |E| List
3. ∀m:Label. Causal(E)(<tr>_m)
4. True
5. tr’ : |E| List
6. tr’ ≤ tr
7. x : |E|
8. (x ∈ tr’)
9. ∀tr’:|E| List.
|  tr’ ≤ <tr>_tag(E)(x) ⇒
|  (∀x∈tr’.(∃y∈tr’.is-send(E)(y) & (y =msg=(E) x)))
⊢  (∃y∈tr’.is-send(E)(y) & (y =msg=(E) x))
| by InstHyp [<tr’>_tag(E)(x)] (-1) THEN Auto
|
|\
| .....antecedent .....
| 1. E : TaggedEventStruct
| 2. tr : |E| List
| 3. ∀m:Label. Causal(E)(<tr>_m)
| 4. True
| 5. tr’ : |E| List
| 6. tr’ ≤ tr
| 7. x : |E|
| 8. (x ∈ tr’)
| 9. ∀tr’:|E| List.
|    tr’ ≤ <tr>_tag(E)(x) ⇒
|    (∀x∈tr’.(∃y∈tr’.is-send(E)(y) & (y =msg=(E) x)))
| ⊢  <tr’>_tag(E)(x) ≤ <tr>_tag(E)(x)
|   by ...
|
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. 1. E : TaggedEventStruct
. 2. tr : |E| List
. 3. ∀m:Label. Causal(E)(<tr>_m)
. 4. True
. 5. tr’ : |E| List
. 6. tr’ ≤ tr
. 7. x : |E|
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. 8. (x ∈ tr')
. 9. ∀tr':|E| List.
. |  tr' ≤ <tr>_tag(E)(x) ⇒
. |  (∀x∈tr'.(∃y∈tr'.is-send(E)(y) & (y =msg=(E) x)))
. 10. (∀x@0∈<tr'>_tag(E)(x).(∃y∈<tr'>_tag(E)(x).is-send(E)(y)
. |   & (y =msg=(E) x@0)))
. ⊢  (∃y∈tr'.is-send(E)(y) & (y =msg=(E) x))
. | by Unfold `l_all` (-1) THEN InstHyp [x] (-1) THENA Auto
. |
. |\
. | .....antecedent .....
. | 1. E : TaggedEventStruct
. | 2. tr : |E| List
. | 3. ∀m:Label. Causal(E)(<tr>_m)
. | 4. True
. | 5. tr' : |E| List
. | 6. tr' ≤ tr
. | 7. x : |E|
. | 8. (x ∈ tr')
. | 9. ∀tr':|E| List.
. |    tr' ≤ <tr>_tag(E)(x) ⇒
. |    (∀x∈tr'.(∃y∈tr'.is-send(E)(y) & (y =msg=(E) x)))
. | 10. ∀x@0:|E|.
. |     (x@0 ∈ <tr'>_tag(E)(x)) ⇒
. |     (∃y∈<tr'>_tag(E)(x).is-send(E)(y) & (y =msg=(E) x@0))
. | ⊢  (x ∈ <tr'>_tag(E)(x))  by RWO "member_tag_sublist" 0 THEN Auto
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. .\
. . 1. E : TaggedEventStruct
. . 2. tr : |E| List
. . 3. ∀m:Label. Causal(E)(<tr>_m)
. . 4. True
. . 5. tr' : |E| List
. . 6. tr' ≤ tr
. . 7. x : |E|
. . 8. (x ∈ tr')
. . 9. ∀tr':|E| List.
. . |  tr' ≤ <tr>_tag(E)(x) ⇒
. . |  (∀x∈tr'.(∃y∈tr'.is-send(E)(y) & (y =msg=(E) x)))
. . 10. ∀x@0:|E|.
. . |   (x@0 ∈ <tr'>_tag(E)(x)) ⇒
. . |   (∃y∈<tr'>_tag(E)(x).is-send(E)(y) & (y =msg=(E) x@0))
. . 11. (∃y∈<tr'>_tag(E)(x).is-send(E)(y) & (y =msg=(E) x))
. . ⊢  (∃y∈tr'.is-send(E)(y) & (y =msg=(E) x))
. . | by RepeatFor 3 (ParallelOp (-1))
. . |
. . 1. E : TaggedEventStruct
. . 2. tr : |E| List
. . 3. ∀m:Label. Causal(E)(<tr>_m)
. . 4. True
. . 5. tr' : |E| List
. . 6. tr' ≤ tr
. . 7. x : |E|
. . 8. (x ∈ tr')
. . 9. ∀tr':|E| List.
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. .    tr' ≤ <tr>_tag(E)(x) ⇒
. .    (∀x∈tr'.(∃y∈tr'.is-send(E)(y) & (y =msg=(E) x)))
. . 10. ∀x@0:|E|.
. .     (x@0 ∈ <tr'>_tag(E)(x)) ⇒
. .     (∃y∈<tr'>_tag(E)(x).is-send(E)(y) & (y =msg=(E) x@0))

. . 11. y : |E|
. . 12. (y ∈ <tr'>_tag(E)(x))
. . ⊢  (y ∈ tr')  by RWO "member_tag_sublist" (-1) THEN Auto
END
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