```
no_DASH_dup_DASH_fusion  THM

⊢  ∀E:TaggedEventStruct, P,I:((|E| List)→Prop).
|  (P refines (Causal(E) ∧ No-dup-deliver(E))) ⇒
|  ((I
|   ∧ No-dup-send(E)
|   ∧ Tag-by-msg(E)
|   ∧ Causal(E)
|   ∧ No-dup-deliver(E)) fuses P)
|  ⇒
|  ((I ∧ No-dup-send(E)) fuses P)
| by Auto
THEN Try (Unfold `trace_property` 0 THEN Auto)

|    THEN Try (Fold `trace_property` 0 THEN Auto)

|    THEN BackThruLemma `tag_by_msg_fusion_lemma` THENA Auto
|
|\
| 1. E : TaggedEventStruct
| 2. P : (|E| List)→Prop
| 3. I : (|E| List)→Prop
| 4. P refines (Causal(E) ∧ No-dup-deliver(E))
| 5. (I
|    ∧ No-dup-send(E)
|    ∧ Tag-by-msg(E)
|    ∧ Causal(E)
|    ∧ No-dup-deliver(E)) fuses P
| ⊢  P refines Causal(E)
|   by RepeatFor 2 (ParallelOp (-2)) THEN ParallelOp (-1)
|
|      THEN Unfold `prop_and` (-1) THEN Reduce (-1) THEN Auto
|
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. 1. E : TaggedEventStruct
. 2. P : (|E| List)→Prop
. 3. I : (|E| List)→Prop
. 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. 5. (I
. |  ∧ No-dup-send(E)
. |  ∧ Tag-by-msg(E)
. |  ∧ Causal(E)
. |  ∧ No-dup-deliver(E)) fuses P
. ⊢  (I ∧ No-dup-send(E) ∧ Tag-by-msg(E)) fuses P
. | by Using [`J`,Causal(E) ∧ No-dup-deliver(E)]
.
. |      (BackThruLemma `fusion_simplification`) THEN Auto
.
. |    THEN Try (Unfold `trace_property` 0 THEN Auto)
.
. |    THEN Try (Fold `trace_property` 0 THEN Auto)
. |
. |
. |\
. | 1. E : TaggedEventStruct
```

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. | 2. P : (|E| List)→Prop
. | 3. I : (|E| List)→Prop
. | 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. | 5. (I
. | |  ∧ No-dup-send(E)
. | |  ∧ Tag-by-msg(E)
. | |  ∧ Causal(E)
. | |  ∧ No-dup-deliver(E)) fuses P
. | ⊢  ((I ∧ No-dup-send(E) ∧ Tag-by-msg(E))
. | |  ∧ Causal(E)
. | |  ∧ No-dup-deliver(E)) fuses P
. | | by BackThruLemma' `fusion_weakening` THEN Auto
. |
. | |    THEN Try (Unfold `trace_property` 0 THEN Auto)
. |
. | |    THEN Try (Fold `trace_property` 0 THEN Auto)
. | |
. | 1. E : TaggedEventStruct
. | 2. P : (|E| List)→Prop
. | 3. I : (|E| List)→Prop
. | 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. | ⊢  ((I ∧ No-dup-send(E) ∧ Tag-by-msg(E))
. |    ∧ Causal(E)
. |    ∧ No-dup-deliver(E))
. |    refines (I
. |            ∧ No-dup-send(E)
. |            ∧ Tag-by-msg(E)
. |            ∧ Causal(E)
. |            ∧ No-dup-deliver(E))
. |   by Unfolds ``tr_refines prop_and`` 0 THEN Reduce 0 THEN Auto
. |
. |      THEN Try (Fold `trace_property` 0 THEN Auto)
. |
. .\
. . 1. E : TaggedEventStruct
. . 2. P : (|E| List)→Prop
. . 3. I : (|E| List)→Prop
. . 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. . 5. (I
. . |  ∧ No-dup-send(E)
. . |  ∧ Tag-by-msg(E)
. . |  ∧ Causal(E)
. . |  ∧ No-dup-deliver(E)) fuses P
. . ⊢  (I ∧ No-dup-send(E) ∧ Tag-by-msg(E)) fuses (Causal(E)
. . |  ∧ No-dup-deliver(E))
. . | by BackThruLemma `fusion_and` THEN Auto
. .
. . |    THEN Try (Unfold `trace_property` 0 THEN Auto)
. .
. . |    THEN Try (Fold `trace_property` 0 THEN Auto)
. . |
. . |\
. . | 1. E : TaggedEventStruct
. . | 2. P : (|E| List)→Prop
. . | 3. I : (|E| List)→Prop
```

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. . | 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. . | 5. (I
. . | |  ∧ No-dup-send(E)
. . | |  ∧ Tag-by-msg(E)
. . | |  ∧ Causal(E)
. . | |  ∧ No-dup-deliver(E)) fuses P

. . | ⊢  (I ∧ No-dup-send(E) ∧ Tag-by-msg(E)) fuses Causal(E)
. . | | by InstLemma ‘causal_fusion‘ [E] THENA Auto
. . |
. . | |    THEN BackThruLemma’ ‘fusion_weakening‘ THEN Auto
. . |
. . | |    THEN Try (Unfold ‘trace_property‘ 0 THEN Auto)
. . |
. . | |    THEN Try (Fold ‘trace_property‘ 0 THEN Auto)
. . | |
. . | |
. . | 1. E : TaggedEventStruct
. . | 2. P : (|E| List)→Prop
. . | 3. I : (|E| List)→Prop
. . | 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. . | 5. (I
. . |    ∧ No-dup-send(E)
. . |    ∧ Tag-by-msg(E)
. . |    ∧ Causal(E)
. . |    ∧ No-dup-deliver(E)) fuses P
. . | ⊢  (I ∧ No-dup-send(E) ∧ Tag-by-msg(E)) refines PTrue
. . |   by Unfolds ‘‘tr_refines PTrue‘‘ 0 THEN Reduce 0 THEN Auto
. . |
. . |      THEN Try (Fold ‘trace_property‘ 0 THEN Auto)
. . |
. . .\
. . . 1. E : TaggedEventStruct
. . . 2. P : (|E| List)→Prop
. . . 3. I : (|E| List)→Prop
. . . 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. . . 5. (I
. . . |  ∧ No-dup-send(E)
. . . |  ∧ Tag-by-msg(E)
. . . |  ∧ Causal(E)
. . . |  ∧ No-dup-deliver(E)) fuses P
. . . ⊢  (I ∧ No-dup-send(E) ∧ Tag-by-msg(E)) fuses No-dup-deliver(E)
. . . | by InstLemma ‘no_dup_fusion‘ [E] THENA Auto
. . .
. . . |    THEN BackThruLemma’ ‘fusion_weakening‘ THEN Auto
. . .
. . . |    THEN Try (Unfold ‘trace_property‘ 0 THEN Auto)
. . .
. . . |    THEN Try (Fold ‘trace_property‘ 0 THEN Auto)
. . . |
. . . 1. E : TaggedEventStruct
. . . 2. P : (|E| List)→Prop
. . . 3. I : (|E| List)→Prop
. . . 4. P refines (Causal(E) ∧ No-dup-deliver(E))
. . . 5. (I
. . .    ∧ No-dup-send(E)
```

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. . .    ∧ Tag-by-msg(E)
. . .    ∧ Causal(E)
. . .    ∧ No-dup-deliver(E)) fuses P
. . . ⊢  (I ∧ No-dup-send(E) ∧ Tag-by-msg(E)) refines Tag-by-msg(E)
. . .   by Unfolds ``tr_refines prop_and`` 0 THEN Reduce 0 THEN Auto
. . .
. . .      THEN Try (Fold `trace_property` 0 THEN Auto)
END
```