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remove_msgs_nil  THM

⊢  ∀A:Type, msg:(A→A→𝔹), L1,L2:A List.
|  (∀x:A. (x ∈ L1) ⇒ (x ∈ L2)) ⇒
|  Refl(A)(msg(_1,_2)) ⇒ (L1 -msg(a,b) L2) = nil
| by Auto THEN Unfold ‘remove_msgs‘ 0
|
1. A : Type
2. msg : A→A→𝔹
3. L1 : A List
4. L2 : A List
5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
6. Refl(A)(msg(_1,_2))
⊢  filter(λa.reduce(λb,y. ¬2msg(a,b)∧2y;true2;L2);L1) = nil
| by InstLemma ‘filter_is_nil‘ [A;λa.reduce(λb,y. ¬2msg(a,b)∧2
|    y;true2;L2);L1] THENA Auto
|
|\
| .....antecedent .....
| 1. A : Type
| 2. msg : A→A→𝔹
| 3. L1 : A List
| 4. L2 : A List
| 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| 6. Refl(A)(msg(_1,_2))
| ⊢  (∀x∈L1.¬(λa.reduce(λb,y. ¬2msg(a,b)∧2y;true2;L2))(x))
| | by Unfolds ‘‘l_all so_apply‘‘ 0 THEN Reduce 0
| |
| 1. A : Type
| 2. msg : A→A→𝔹
| 3. L1 : A List
| 4. L2 : A List
| 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| 6. Refl(A)(msg(_1,_2))
| ⊢  ∀x:A. (x ∈ L1) ⇒ ¬reduce(λb,y. ¬2msg(x,b)∧2y;true2;L2)
| | by RWO "l_all_reduce<" 0 THENA Auto
| |
| 1. A : Type
| 2. msg : A→A→𝔹
| 3. L1 : A List
| 4. L2 : A List
| 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| 6. Refl(A)(msg(_1,_2))
| ⊢  ∀x:A. (x ∈ L1) ⇒ ¬(∀b∈L2.¬2msg(x,b))
| | by Unfold ‘l_all‘ 0 THEN Auto
| |
| 1. A : Type
| 2. msg : A→A→𝔹
| 3. L1 : A List
| 4. L2 : A List
| 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| 6. Refl(A)(msg(_1,_2))
| 7. x : A
| 8. (x ∈ L1)
| ⊢  ¬(∀b:A. (b ∈ L2) ⇒ ¬2msg(x,b))  by D 0 THEN Auto
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| |
| 1. A : Type
| 2. msg : A→A→𝔹
| 3. L1 : A List
| 4. L2 : A List
| 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| 6. Refl(A)(msg(_1,_2))
| 7. x : A
| 8. (x ∈ L1)
| 9. ∀b:A. (b ∈ L2) ⇒ ¬2msg(x,b)
| ⊢  False  by InstHyp [x] (-1) THEN Auto
| |
| |\
| | .....antecedent .....
| | 1. A : Type
| | 2. msg : A→A→𝔹
| | 3. L1 : A List
| | 4. L2 : A List
| | 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| | 6. Refl(A)(msg(_1,_2))
| | 7. x : A
| | 8. (x ∈ L1)
| | 9. ∀b:A. (b ∈ L2) ⇒ ¬2msg(x,b)
| | ⊢  (x ∈ L2)  by Easy
| |
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| . 1. A : Type
| . 2. msg : A→A→𝔹
| . 3. L1 : A List
| . 4. L2 : A List
| . 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| . 6. Refl(A)(msg(_1,_2))
| . 7. x : A
| . 8. (x ∈ L1)
| . 9. ∀b:A. (b ∈ L2) ⇒ ¬2msg(x,b)
| . 10. ¬2msg(x,x)
| . ⊢  False  by RW assert_pushdownC (-1) THENA Auto THEN D (-1)
| . |
| . 1. A : Type
| . 2. msg : A→A→𝔹
| . 3. L1 : A List
| . 4. L2 : A List
| . 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
| . 6. Refl(A)(msg(_1,_2))
| . 7. x : A
| . 8. (x ∈ L1)
| . 9. ∀b:A. (b ∈ L2) ⇒ ¬2msg(x,b)
| . ⊢  msg(x,x)
| .   by AllHyps (\i. Unfold `refl` i THEN BackThruHyp i THEN Auto)
|
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. 1. A : Type
. 2. msg : A→A→𝔹
. 3. L1 : A List
. 4. L2 : A List
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. 5. ∀x:A. (x ∈ L1) ⇒ (x ∈ L2)
. 6. Refl(A)(msg(_1,_2))
. 7. filter(λa.reduce(λb,y. ¬₂msg(a,b)∧₂y;true₂;L2);L1) ~ nil
. ⊢  filter(λa.reduce(λb,y. ¬₂msg(a,b)∧₂y;true₂;L2);L1) = nil
.   by HypSubst (-1) 0 THEN Auto
END
```