```
totalorder_switchable0  THM

⊢  ∀E:EventStruct. switchable0(E)(totalorder(E))
| by Auto THEN
Subst
|    totalorder(E)
|    =
|    local_deliver_property(E;λf.∀p,q:Label.
|    agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))) 0

|    THEN Auto
|
|\
| .....equality .....
| 1. E : EventStruct
| ⊢  totalorder(E)
|    =
|    local_deliver_property(E;λf.∀p,q:Label.
|    agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q))))
|   by Ext THEN Auto THEN Try (Fold ‘trace_property‘ 0 THEN Auto)
|
|      ...
| THEN Auto
|
.\
. 1. E : EventStruct
. ⊢  switchable0(E)
. |  (local_deliver_property(E;λf.∀p,q:Label.
. |   agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))))
. | by ...
. |
. |\
. | 1. E : EventStruct
. | 2. f : Label→(|E| List)
. | 3. g : Label→(|E| List)
. | 4. ∀p:Label. g(p) ≤ f(p)
. | 5. ∀p,q:Label.
. | |  agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | 6. p : Label
. | 7. q : Label
. | ⊢  agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. | | by InstHyp [p] 4 THENA Auto THEN InstHyp [q
. | |    ] 4 THENA Auto THEN Thin 4
. | THEN InstHyp [p;q
. | |    ] 4 THENA Auto THEN Thin 4
. | |
. | 1. E : EventStruct
. | 2. f : Label→(|E| List)
. | 3. g : Label→(|E| List)
. | 4. p : Label
. | 5. q : Label
. | 6. g(p) ≤ f(p)
. | 7. g(q) ≤ f(q)
. | 8. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | ⊢  agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
```

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. |   by BackThruLemma' `agree_on_common_iseg` THEN Auto
. |
. |      THEN BackThruLemma `iseg_map` THEN Auto
. |
. |\
. | 1. E : EventStruct
. | 2. f : Label→(|E| List)
. | 3. g : Label→(|E| List)
. | 4. ∃a:|E|. ∀p:Label. g(p) = filter(λb.¬2(b =msg=(E) a);f(p))
. | 5. ∀p,q:Label.
. | |  agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | 6. p : Label
. | 7. q : Label
. | ⊢  agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. | | by ExRepD
. | THEN InstHyp [p
. | |    ] (-4) THENA Auto THEN HypSubst (-1) 0 THENA Auto
. |
. | |    THEN Thin (-1)
. | THEN InstHyp [q
. | |    ] (-4) THENA Auto THEN HypSubst (-1) 0 THENA Auto
. |
. | |    THEN Thin (-1)
. | THEN InstHyp [p;q] (-3) THENA Auto
. | |
. | 1. E : EventStruct
. | 2. f : Label→(|E| List)
. | 3. g : Label→(|E| List)
. | 4. a : |E|
. | 5. ∀p:Label. g(p) = filter(λb.¬2(b =msg=(E) a);f(p))
. | 6. ∀p,q:Label.
. | |  agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | 7. p : Label
. | 8. q : Label
. | 9. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | ⊢  agree_on_common(|MS(E)|;map(msg(E);filter(λb.
. | |  ¬2(b =msg=(E) a);f(p)));map(msg(E);filter(λb.
. | |  ¬2(b =msg=(E) a);f(q))))
. | | by Subst (λb.¬2(b =msg=(E) a))
. | |          =
. | |          (λb.¬2(b =(MS(E)) (msg(E)(a)))) o msg(E) 0
. | THENA Auto
. | |
. | |\
. | | .....equality .....
. | | 1. E : EventStruct
. | | 2. f : Label→(|E| List)
. | | 3. g : Label→(|E| List)
. | | 4. a : |E|
. | | 5. ∀p:Label. g(p) = filter(λb.¬2(b =msg=(E) a);f(p))
. | | 6. ∀p,q:Label.
. | |    agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | | 7. p : Label
. | | 8. q : Label
. | | 9. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
```

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. | | ⊢  (λb.¬2(b =msg=(E) a))
. | |    =
. | |    (λb.¬2(b =(MS(E)) (msg(E)(a)))) o msg(E)
. | |   by ...
. | |
. | .\
. | . 1. E : EventStruct
. | . 2. f : Label→(|E| List)
. | . 3. g : Label→(|E| List)
. | . 4. a : |E|
. | . 5. ∀p:Label. g(p) = filter(λb.¬2(b =msg=(E) a);f(p))
. | . 6. ∀p,q:Label.
. | . |  agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | . 7. p : Label
. | . 8. q : Label
. | . 9. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | . ⊢  agree_on_common(|MS(E)|;map(msg(E);filter(...
. | . |   o msg(E);f
. | . |            (p)));map(msg(E);filter((λb.
. | . |                                         ¬2(b
. | . |                                            =(MS(E)) (msg(E)(a))))
. | . |   o msg(E);f(q))))
. | . | by InstLemma ‘filter_map‘ [|E|;|MS(E)|;msg(E);λb.
. | . |    ¬2(b =(MS(E)) (msg(E)(a)));f(p)] THENA Auto
. | .
. | . |    THEN RevHypSubst (-1) 0 THENA Auto
. | .
. | . |    THEN InstLemma ‘filter_map‘ [|E|;|MS(E)|;msg(E);λb.
. | . |    ¬2(b =(MS(E)) (msg(E)(a)));f(q)] THENA Auto
. | .
. | . |    THEN RevHypSubst (-1) 0 THENA Auto
. | . |
. | . 1. E : EventStruct
. | . 2. f : Label→(|E| List)
. | . 3. g : Label→(|E| List)
. | . 4. a : |E|
. | . 5. ∀p:Label. g(p) = filter(λb.¬2(b =msg=(E) a);f(p))
. | . 6. ∀p,q:Label.
. | .    agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | . 7. p : Label
. | . 8. q : Label
. | . 9. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. | . 10. filter(λb.¬2(b =(MS(E)) (msg(E)(a)));map(msg(E);f(p)))
. | .     =
. | .     map(msg(E);filter((λb.¬2(b =(MS(E)) (msg(E)(a))))
. | .      o msg(E);f(p)))
. | . 11. filter(λb.¬2(b =(MS(E)) (msg(E)(a)));map(msg(E);f(q)))
. | .     =
. | .     map(msg(E);filter((λb.¬2(b =(MS(E)) (msg(E)(a))))
. | .      o msg(E);f(q)))
. | . ⊢  agree_on_common(|MS(E)|;filter(λb.
. | .    ¬2(b =(MS(E)) (msg(E)(a)));map(msg(E);f(p)));filter(λb.
. | .    ¬2(b =(MS(E)) (msg(E)(a)));map(msg(E);f(q))))
. | .   by BackThruLemma ‘agree_on_common_filter‘ THEN  Auto
. |
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. .\
. . 1. E : EventStruct
. . 2. f : Label→(|E| List)
. . 3. g : Label→(|E| List)
. . 4. h : Label→(|E| List)
. . 5. ∀p,q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . 6. ∀p:Label. h(p) = ((f(p)) @ (g(p)))
. . 7. ∀p,q:Label.
. . |  agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . 8. ∀p,q:Label.
. . |  agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . 9. p : Label
. . 10. q : Label
. . ⊢  agree_on_common(|MS(E)|;map(msg(E);h(p));map(msg(E);h(q)))
. . | by AllHyps (InstHyp [p]) THENA Auto
. . THEN AllHyps (InstHyp [q
. . |    ]) THENA Auto
. . THEN SubstFor h(p) 0 THENA Auto'
. .
. . |    THEN SubstFor h(q) 0 THENA Auto'
. . |
. . 1. E : EventStruct
. . 2. f : Label→(|E| List)
. . 3. g : Label→(|E| List)
. . 4. h : Label→(|E| List)
. . 5. ∀p,q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . 6. ∀p:Label. h(p) = ((f(p)) @ (g(p)))
. . 7. ∀p,q:Label.
. . |  agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . 8. ∀p,q:Label.
. . |  agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . 9. p : Label
. . 10. q : Label
. . 11. ∀q:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . 12. ∀q:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . 13. h(p) = ((f(p)) @ (g(p)))
. . 14. ∀q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . 15. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . 16. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . 17. agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . 18. ∀q@0:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);g(q));map(msg(E);g(q@0)))
. . 19. ∀q@0:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);f(q));map(msg(E);f(q@0)))
. . 20. h(q) = ((f(q)) @ (g(q)))
. . 21. ∀q@0:Label. (∀x∈f(q).(∀y∈g(q@0).¬(x =msg=(E) y)))
. . ⊢  agree_on_common(|MS(E)|;map(msg(E);(f(p))
. . |  @ (g(p)));map(msg(E);(f(q)) @ (g(q))))
. . | by RW MapAppendC 0 THENA Auto
. . |
. . 1. E : EventStruct
. . 2. f : Label→(|E| List)
. . 3. g : Label→(|E| List)
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. . 4. h : Label→(|E| List)
. . 5. ∀p,q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . 6. ∀p:Label. h(p) = ((f(p)) @ (g(p)))
. . 7. ∀p,q:Label.
. . |  agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . 8. ∀p,q:Label.
. . |  agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . 9. p : Label
. . 10. q : Label
. . 11. ∀q:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . 12. ∀q:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . 13. h(p) = ((f(p)) @ (g(p)))
. . 14. ∀q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . 15. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . 16. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . 17. agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . 18. ∀q@0:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);g(q));map(msg(E);g(q@0)))
. . 19. ∀q@0:Label.
. . |   agree_on_common(|MS(E)|;map(msg(E);f(q));map(msg(E);f(q@0)))
. . 20. h(q) = ((f(q)) @ (g(q)))
. . 21. ∀q@0:Label. (∀x∈f(q).(∀y∈g(q@0).¬(x =msg=(E) y)))
. . ⊢  agree_on_common(|MS(E)|;map(msg(E);f(p))
. . |  @ map(msg(E);g(p));map(msg(E);f(q)) @ map(msg(E);g(q)))
. . | by BackThruLemma ‘agree_on_common_append‘ THEN Auto
. . |
. . |\
. . | 1. E : EventStruct
. . | 2. f : Label→(|E| List)
. . | 3. g : Label→(|E| List)
. . | 4. h : Label→(|E| List)
. . | 5. ∀p,q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . | 6. ∀p:Label. h(p) = ((f(p)) @ (g(p)))
. . | 7. ∀p,q:Label.
. . |    agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . | 8. ∀p,q:Label.
. . |    agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . | 9. p : Label
. . | 10. q : Label
. . | 11. ∀q:Label.
. . |     agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . | 12. ∀q:Label.
. . |     agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . | 13. h(p) = ((f(p)) @ (g(p)))
. . | 14. ∀q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . | 15. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . | 16. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . | 17. agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . | 18. ∀q@0:Label.
. . |     agree_on_common(|MS(E)|;map(msg(E);g(q));map(msg(E);g(q@0)))
. . | 19. ∀q@0:Label.
. . |     agree_on_common(|MS(E)|;map(msg(E);f(q));map(msg(E);f(q@0)))
. . | 20. h(q) = ((f(q)) @ (g(q)))
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. . | 21. ∀q@0:Label. (∀x∈f(q).(∀y∈g(q@0).¬(x =msg=(E) y)))
. . | ⊢  (∀x∈map(msg(E);f(q)).¬(x ∈ map(msg(E);g(p))))
. . |   by All (Unfold ‘l_all‘) THEN Auto THEN D 0 THENA Auto
. . |
. . |      THEN AllHyps (RWO "member_map") THENA Auto
. . | THEN ExRepD
. . |
. . |      THEN InstHyp [q;p;y1;y] 5 THENA Auto
. . | THEN D (-1)
. . |
. . |      THEN Unfold ‘event_msg_eq‘ 0 THEN Reduce 0
. . | THEN SubstFor
. . |      msg(E)(y1) 0
. . | THEN SubstFor msg(E)(y) 0
. . |
. . |      THEN InstLemma ‘msg_eq_equiv‘ [MS(E)] THENA Auto
. . |
. . |      THEN UseEquiv
. . |
. . .\
. . . 1. E : EventStruct
. . . 2. f : Label→(|E| List)
. . . 3. g : Label→(|E| List)
. . . 4. h : Label→(|E| List)
. . . 5. ∀p,q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . . 6. ∀p:Label. h(p) = ((f(p)) @ (g(p)))
. . . 7. ∀p,q:Label.
. . .    agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . . 8. ∀p,q:Label.
. . .    agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . . 9. p : Label
. . . 10. q : Label
. . . 11. ∀q:Label.
. . .     agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . . 12. ∀q:Label.

. . .     agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . . 13. h(p) = ((f(p)) @ (g(p)))
. . . 14. ∀q:Label. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . . 15. (∀x∈f(p).(∀y∈g(q).¬(x =msg=(E) y)))
. . . 16. agree_on_common(|MS(E)|;map(msg(E);f(p));map(msg(E);f(q)))
. . . 17. agree_on_common(|MS(E)|;map(msg(E);g(p));map(msg(E);g(q)))
. . . 18. ∀q@0:Label.
. . .     agree_on_common(|MS(E)|;map(msg(E);g(q));map(msg(E);g(q@0)))
. . . 19. ∀q@0:Label.
. . .     agree_on_common(|MS(E)|;map(msg(E);f(q));map(msg(E);f(q@0)))
. . . 20. h(q) = ((f(q)) @ (g(q)))
. . . 21. ∀q@0:Label. (∀x∈f(q).(∀y∈g(q@0).¬(x =msg=(E) y)))
. . . ⊢  (∀x∈map(msg(E);f(p)).¬(x ∈ map(msg(E);g(q))))
. . .   by All (Unfold ‘l_all‘) THEN Auto THEN D 0 THENA Auto
. . .
. . .      THEN AllHyps (RWO "member_map") THENA Auto
. . . THEN ExRepD
. . .
. . .      THEN InstHyp [p;q;y1;y] 5 THENA Auto
. . . THEN D (-1)
```

```
. . .
. . .      THEN Unfold `event_msg_eq` 0 THEN Reduce 0
. . . THEN SubstFor
. . .      msg(E)(y1) 0
. . . THEN SubstFor msg(E)(y) 0
. . .
. . .      THEN InstLemma `msg_eq_equiv` [MS(E)] THENA Auto
. . .
. . .      THEN UseEquiv
END
```