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cons_member  THM

⊢  ∀T:Type, l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
| by Unfold ‘l_member‘ 0 THEN Auto
|
|\
| 1. T : Type
| 2. l : T List
| 3. a : T
| 4. x : T
| 5. ∃i:ℕ. i<||[a / l]|| & x = [a / l][i]
| ⊢  x = a ∨ (∃i:ℕ. i<||l|| & x = l[i])
| | by ExRepD THEN MoveToConcl (-1) THEN MoveToConcl (-1)
|
| |    THEN CaseNatZero ‘i’
| |
| |
| |\
| | 1. T : Type
| | 2. l : T List
| | 3. a : T
| | 4. x : T
| | 5. i : ℕ
| | 6. i = 0
| | ⊢  0<||[a / l]||
| |    ⇒
| |    x = [a / l][0] ⇒ x = a ∨ (∃i:ℕ. i<||l|| & x = l[i])
| |   by Reduce 0 THEN Auto THEN Sel 1 (D 0) THEN Auto
| |
| .\
| . 1. T : Type
| . 2. l : T List
| . 3. a : T
| . 4. x : T
| . 5. i : ℕ
| . 6. i ≠ 0
| . 7. 0<i
| . ⊢  i<||[a / l]||
| . |  ⇒
| . |  x = [a / l][i] ⇒ x = a ∨ (∃i:ℕ. i<||l|| & x = l[i])
| . | by Reduce 0 THEN D 0 THENA Auto
| . |
| . 1. T : Type
| . 2. l : T List
| . 3. a : T
| . 4. x : T
| . 5. i : ℕ
| . 6. i ≠ 0
| . 7. 0<i
| . 8. i<||l||+1
| . ⊢  x = [a / l][i] ⇒ x = a ∨ (∃i:ℕ. i<||l|| & x = l[i])
| . | by RWW "select_cons_tl" 0 THEN Auto
| . |
| . 1. T : Type
| . 2. l : T List
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| . 3. a : T
| . 4. x : T
| . 5. i : ℕ
| . 6. i ≠ 0
| . 7. 0<i
| . 8. i<||l||+1
| . 9. x = l[(i-1)]
| . ⊢  x = a ∨ (∃i:ℕ. i<||l|| & x = l[i])
| .   by Sel 2 (D 0) THENA Auto
| . THEN InstConcl [i-1]
| . THEN Auto
|
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. 1. T : Type
. 2. l : T List
. 3. a : T
. 4. x : T
. 5. x = a ∨ (∃i:ℕ. i<||l|| & x = l[i])
. ⊢  ∃i:ℕ. i<||[a / l]|| & x = [a / l][i]  by D (-1)
. |
. |\
. | 1. T : Type
. | 2. l : T List
. | 3. a : T
. | 4. x : T
. | 5. x = a
. | ⊢  ∃i:ℕ. i<||[a / l]|| & x = [a / l][i]
. |   by InstConcl [0] THEN Reduce 0 THEN Auto'
. |
. .\
. . 1. T : Type
. . 2. l : T List
. . 3. a : T
. . 4. x : T
. . 5. ∃i:ℕ. i<||l|| & x = l[i]
. . ⊢  ∃i:ℕ. i<||[a / l]|| & x = [a / l][i]
. . | by ExRepD THEN InstConcl [i+1] THEN Reduce 0 THEN Auto'
. . |
. . 1. T : Type
. . 2. l : T List
. . 3. a : T
. . 4. x : T
. . 5. i : ℕ
. . 6. i<||l||
. . 7. x = l[i]
. . ⊢  x = [a / l][(i+1)]  by RWW "select_cons_tl" 0 THEN Auto'
. . |

. . 1. T : Type
. . 2. l : T List
. . 3. T
. . 4. x : T
. . 5. i : ℕ
. . 6. i<||l||
. . 7. x = l[i]
. . ⊢  x = l[(i+1-1)]  by ArithSimp 0 THEN Auto
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END
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