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member_filter  THM

⊢  ∀T:Type, P:(T→𝔹), L:T List, x:T.
|  (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
| by RepeatFor 3 (D 0 THENA Auto) THEN ListInd (-1) THEN Reduce 0
|
|\
| 1. T : Type
| 2. P : T→𝔹
| 3. T List
| ⊢  ∀x:T. (x ∈ nil) ⟺ (x ∈ nil) & P(x)
|   by RWO "nil_member" 0 THEN Auto
|
.\
. 1. T : Type
. 2. P : T→𝔹
. 3. T List
. 4. u : T
. 5. v : T List
. 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. ⊢  ∀x:T.
. |  (x ∈ if P(u)→ [u / filter(P;v)] else filter(P;v) fi)
. |  ⟺
. |  (x ∈ [u / v]) & P(x)
. | by SplitOnConclITE THENA Auto THEN RWO "cons_member" 0 THEN Auto
. |
. |\
. | 1. T : Type
. | 2. P : T→𝔹
. | 3. T List
. | 4. u : T
. | 5. v : T List
. | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | 7. P(u)
. | 8. x@0 : T
. | 9. x@0 = u ∨ (x@0 ∈ filter(P;v))
. | ⊢  x@0 = u ∨ (x@0 ∈ v)  by ParallelOp (-1) THEN Auto
. | |
. | 1. T : Type
. | 2. P : T→𝔹
. | 3. T List
. | 4. u : T
. | 5. v : T List
. | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | 7. P(u)
. | 8. x@0 : T
. | 9. (x@0 ∈ filter(P;v))
. | ⊢  (x@0 ∈ v)
. |   by InstHyp [x@0] (-4) THEN Auto THEN ThinTrivial THEN Auto
. |
. |\
. | 1. T : Type
. | 2. P : T→𝔹
. | 3. T List
. | 4. u : T
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. | 5. v : T List
. | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | 7. P(u)
. | 8. x@0 : T
. | 9. x@0 = u ∨ (x@0 ∈ filter(P;v))
. | ⊢  P(x@0)  by D (-1)
. | |
. | |\
. | | 1. T : Type
. | | 2. P : T→𝔹
. | | 3. T List
. | | 4. u : T
. | | 5. v : T List
. | | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | | 7. P(u)
. | | 8. x@0 : T
. | | 9. x@0 = u
. | | ⊢  P(x@0)  by HypSubst (-1) 0 THEN Auto
. | |
. | .\
. | . 1. T : Type
. | . 2. P : T→𝔹
. | . 3. T List
. | . 4. u : T
. | . 5. v : T List
. | . 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | . 7. P(u)
. | . 8. x@0 : T
. | . 9. (x@0 ∈ filter(P;v))
. | . ⊢  P(x@0)
. | .   by InstHyp [x@0] (-4) THEN Auto THEN ThinTrivial THEN Auto
. |
. |\
. | 1. T : Type
. | 2. P : T→𝔹
. | 3. T List
. | 4. u : T
. | 5. v : T List
. | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | 7. P(u)
. | 8. x@0 : T
. | 9. x@0 = u ∨ (x@0 ∈ v)
. | 10. P(x@0)
. | ⊢  x@0 = u ∨ (x@0 ∈ filter(P;v))  by ParallelOp (-2) THEN Auto
. | |
. | 1. T : Type
. | 2. P : T→𝔹
. | 3. T List
. | 4. u : T
. | 5. v : T List
. | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | 7. P(u)
. | 8. x@0 : T
. | 9. (x@0 ∈ v)
. | 10. P(x@0)
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. | ⊢  (x@0 ∈ filter(P;v))  by EasyHyp
. |
. |\
. | 1. T : Type
. | 2. P : T→𝔹
. | 3. T List
. | 4. u : T
. | 5. v : T List
. | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | 7. ¬P(u)
. | 8. x : T
. | 9. (x ∈ filter(P;v))
. | ⊢  x = u ∨ (x ∈ v)
. |   by InstHyp [x
. |      ] (-4) THEN Auto THEN ThinTrivial THEN Auto THEN SimpConcl
. |
. |\
. | 1. T : Type
. | 2. P : T→𝔹
. | 3. T List
. | 4. u : T
. | 5. v : T List
. | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. | 7. ¬P(u)
. | 8. x : T
. | 9. (x ∈ filter(P;v))
. | ⊢  P(x)
. |   by InstHyp [x
. |      ] (-4) THEN Auto THEN ThinTrivial THEN Auto THEN SimpConcl
. |
. .\
. . 1. T : Type
. . 2. P : T→𝔹
. . 3. T List
. . 4. u : T
. . 5. v : T List
. . 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. . 7. ¬P(u)
. . 8. x : T
. . 9. x = u ∨ (x ∈ v)
. . 10. P(x)
. . ⊢  (x ∈ filter(P;v))  by D (-2)
. . |
. . |\
. . | 1. T : Type
. . | 2. P : T→𝔹
. . | 3. T List
. . | 4. u : T
. . | 5. v : T List
. . | 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. . | 7. ¬P(u)
. . | 8. x : T
. . | 9. x = u
. . | 10. P(x)
. . | ⊢  (x ∈ filter(P;v))
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. . |   by HypSubst (-2) (-1) THEN Auto THEN D (-4) THEN Auto
. . |
. . .\
. . . 1. T : Type
. . . 2. P : T→ℙ
. . . 3. T List
. . . 4. u : T
. . . 5. v : T List
. . . 6. ∀x:T. (x ∈ filter(P;v)) ⟺ (x ∈ v) & P(x)
. . . 7. ¬P(u)
. . . 8. x : T
. . . 9. (x ∈ v)
. . . 10. P(x)
. . . ⊢  (x ∈ filter(P;v))  by Easy
END
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