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member_map  THM

⊢  ∀T,T':Type, a:T List, x:T', f:(T→T').
|  (x ∈ map(f;a)) ⟺ (∃y:T. (y ∈ a) & x = f(y))
| by Unfold `l_member` 0 THEN UnivCD THEN Auto
|
|\
| 1. T : Type
| 2. T' : Type
| 3. a : T List
| 4. x : T'
| 5. f : T→T'
| 6. ∃i:ℕ. i<||map(f;a)|| & x = map(f;a)[i]
| ⊢  ∃y:T. (∃i:ℕ. i<||a|| & y = a[i]) & x = f(y)
| | by ExRepD THEN
| RWW "map_length" (-2) THENA Auto THEN
|
| |    RWW "map_select" (-1) THENA Auto
| |
| 1. T : Type
| 2. T' : Type
| 3. a : T List
| 4. x : T'
| 5. f : T→T'
| 6. i : ℕ
| 7. i<||a||
| 8. x = f(a[i])
| ⊢  ∃y:T. (∃i:ℕ. i<||a|| & y = a[i]) & x = f(y)
| | by InstConcl [a[i]] THEN Auto
| |
| 1. T : Type
| 2. T' : Type
| 3. a : T List
| 4. x : T'
| 5. f : T→T'
| 6. i : ℕ
| 7. i<||a||
| 8. x = f(a[i])
| ⊢  ∃i@0:ℕ. i@0<||a|| & a[i] = a[i@0]  by InstConcl [i] THEN Auto
|
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. 1. T : Type
. 2. T' : Type
. 3. a : T List
. 4. x : T'
. 5. f : T→T'
. 6. ∃y:T. (∃i:ℕ. i<||a|| & y = a[i]) & x = f(y)
. ⊢  ∃i:ℕ. i<||map(f;a)|| & x = map(f;a)[i]
. | by ExRepD THEN InstConcl [i] THEN Auto'
. |
. 1. T : Type
. 2. T' : Type
. 3. a : T List
. 4. x : T'
. 5. f : T→T'
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. 6. y : T
. 7. i : ℕ
. 8. i<||a||
. 9. y = a[i]
. 10. x = f(y)
. ⊢  x = map(f;a)[i]  by RWO "map_select" 0 THEN Auto
. |
. 1. T : Type
. 2. T' : Type
. 3. a : T List
. 4. x : T'
. 5. f : T⟶T'
. 6. y : T
. 7. i : ℕ
. 8. i<||a||
. 9. y = a[i]
. 10. x = f(y)
. ⊢  x = f(a[i])  by RevHypSubst (-2) 0 THEN Auto
END
```