```
compose_bij  THM

⊢  ∀T:Type, f,g:(T→T).
|  Bij(T; T; f) ⇒ Bij(T; T; g) ⇒ Bij(T; T; f o g)
| by ...
Auto
|
|\
| 1. T : Type
| 2. f : T→T
| 3. g : T→T
| 4. ∀a1,a2:T. f(a1) = f(a2) ⇒ a1 = a2
| 5. ∀b:T. ∃a:T. f(a) = b
| 6. ∀a1,a2:T. g(a1) = g(a2) ⇒ a1 = a2
| 7. ∀b:T. ∃a:T. g(a) = b
| 8. a1 : T
| 9. a2 : T
| 10. f(g(a1)) = f(g(a2))
| ⊢  a1 = a2  by BackThruHyp (-5) THENA Auto THEN EasyHyp
|
.\
. 1. T : Type
. 2. f : T→T
. 3. g : T→T
. 4. ∀a1,a2:T. f(a1) = f(a2) ⇒ a1 = a2
. 5. ∀b:T. ∃a:T. f(a) = b
. 6. ∀a1,a2:T. g(a1) = g(a2) ⇒ a1 = a2
. 7. ∀b:T. ∃a:T. g(a) = b
. 8. b : T
. ⊢  ∃a:T. f(g(a)) = b  by InstHyp [b] (-4) THENA Auto THEN ExRepD
. |
. 1. T : Type
. 2. f : T→T
. 3. g : T→T
. 4. ∀a1,a2:T. f(a1) = f(a2) ⇒ a1 = a2
. 5. ∀b:T. ∃a:T. f(a) = b
. 6. ∀a1,a2:T. g(a1) = g(a2) ⇒ a1 = a2
. 7. ∀b:T. ∃a:T. g(a) = b
. 8. b : T
. 9. a : T
. 10. f(a) = b
. ⊢  ∃a:T. f(g(a)) = b  by InstHyp [a] (-4) THENA Auto THEN ExRepD
. |
. 1. T : Type
. 2. f : T→T
. 3. g : T→T
. 4. ∀a1,a2:T. f(a1) = f(a2) ⇒ a1 = a2
. 5. ∀b:T. ∃a:T. f(a) = b
. 6. ∀a1,a2:T. g(a1) = g(a2) ⇒ a1 = a2
. 7. ∀b:T. ∃a:T. g(a) = b
. 8. b : T
. 9. a : T
. 10. f(a) = b
. 11. a@0 : T
. 12. g(a@0) = a
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. ⊢  ∃a:T. f(g(a)) = b  by InstConcl [a@0] THEN Auto
. |
. 1. T : Type
. 2. f : T→T
. 3. g : T→T
. 4. ∀a1,a2:T. f(a1) = f(a2) ⇒ a1 = a2
. 5. ∀b:T. ∃a:T. f(a) = b
. 6. ∀a1,a2:T. g(a1) = g(a2) ⇒ a1 = a2
. 7. ∀b:T. ∃a:T. g(a) = b
. 8. b : T
. 9. a : T
. 10. f(a) = b
. 11. a@0 : T
. 12. g(a@0) = a
. ⊢  f(g(a@0)) = b  by HypSubst (-1) 0 THEN Auto
END
```