```
permute_sum  THM

⊢  ∀n:ℕ, f:(ℕn→ℤ), p:(ℕn→ℕn).
|  Bij(ℕn; ℕn; p) ⇒ sum(f(p(x)) | x < n) = sum(f(x) | x < n)
| by Auto THEN FwdThruLemma ‘permute_by_flips‘ [(-1)] THENA Auto

|    THEN ExRepD THEN HypSubst (-1) 0 THENA Auto THEN AllHyps Thin
|
1. n : ℕ
2. f : ℕn→ℤ
3. L : ℕ(n-1) List
⊢  sum(f(compose_flips(L)(x)) | x < n) = sum(f(x) | x < n)
| by MoveToConcl (-2) THEN ListInd (-1)
|
|\
| 1. n : ℕ
| 2. ℕ(n-1) List
| ⊢  ∀f:(ℕn→ℤ).
|    sum(f(compose_flips(nil)(x)) | x < n) = sum(f(x) | x < n)
|   by Unfold ‘compose_flips‘ 0 THEN Reduce 0 THEN Auto
|
.\
. 1. n : ℕ
. 2. ℕ(n-1) List
. 3. u : ℕ(n-1)
. 4. v : ℕ(n-1) List
. 5. ∀f:(ℕn→ℤ).
. |  sum(f(compose_flips(v)(x)) | x < n) = sum(f(x) | x < n)
. ⊢  ∀f:(ℕn→ℤ).
. |  sum(f(compose_flips([u / v])(x)) | x < n) = sum(f(x) | x < n)
. | by Assert ∀f:(ℕn→ℤ).
. |    sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. |
. |\
. | .....assertion .....
. | 1. n : ℕ
. | 2. ℕ(n-1) List
. | 3. ℕ(n-1)
. | 4. v : ℕ(n-1) List
. | 5. ∀f:(ℕn→ℤ).
. |    sum(f(compose_flips(v)(x)) | x < n) = sum(f(x) | x < n)
. | ⊢  ∀f:(ℕn→ℤ).
. |    sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. |   by ...
. |
. .\
. . 1. n : ℕ
. . 2. ℕ(n-1) List
. . 3. u : ℕ(n-1)
. . 4. v : ℕ(n-1) List
. . 5. ∀f:(ℕn→ℤ).
. . |  sum(f(compose_flips(v)(x)) | x < n) = sum(f(x) | x < n)
. . 6. ∀f:(ℕn→ℤ).
. . |  sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . ⊢  ∀f:(ℕn→ℤ).
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. . |  sum(f(compose_flips([u / v])(x)) | x < n) = sum(f(x) | x < n)
. . | by Thin (-2) THEN ReduceSOAps 0
. . |
. . 1. n : ℕ
. . 2. ℕ(n-1) List
. . 3. u : ℕ(n-1)
. . 4. v : ℕ(n-1) List
. . 5. ∀f:(ℕn→ℤ).
. . |  sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . ⊢  ∀f:(ℕn→ℤ).
. . |  sum(f(compose_flips([u / v])(x)) | x < n) = sum(f(x) | x < n)
. . | by Auto THEN InstHyp [f o (u, u+1)] (-2)
. . |
. . |\
. . | .....wf .....
. . | 1. n : ℕ
. . | 2. ℕ(n-1) List
. . | 3. u : ℕ(n-1)
. . | 4. v : ℕ(n-1) List
. . | 5. ∀f:(ℕn→ℤ).
. . |    sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . | 6. f : ℕn→ℤ
. . | ⊢  f o (u, u+1) ∈ ℕn→ℤ  by Using [‘B’,ℕn] Auto
. . |
. . .\
. . . 1. n : ℕ
. . . 2. ℕ(n-1) List
. . . 3. u : ℕ(n-1)
. . . 4. v : ℕ(n-1) List
. . . 5. ∀f:(ℕn→ℤ).
. . . |  sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . . 6. f : ℕn→ℤ
. . . 7. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . |  =
. . . |  sum((f o (u, u+1))(x) | x < n)
. . . ⊢  sum(f(compose_flips([u / v])(x)) | x < n) = sum(f(x) | x < n)
. . . | by Assert sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . |           =
. . . |           sum(f(compose_flips([u / v])(x)) | x < n)
. . . |
. . . |\
. . . | .....assertion .....
. . . | 1. n : ℕ
. . . | 2. ℕ(n-1) List
. . . | 3. u : ℕ(n-1)
. . . | 4. v : ℕ(n-1) List
. . . | 5. ∀f:(ℕn→ℤ).
. . . | |  sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . . | 6. f : ℕn→ℤ
. . . | 7. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . | |  =
. . . | |  sum((f o (u, u+1))(x) | x < n)
. . . | ⊢  sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . | |  =
. . . | |  sum(f(compose_flips([u / v])(x)) | x < n)
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. . . | | by BackThruLemma `sum_functionality` THEN Using [`B',ℕn
. . . | |    ] Auto
. . . |
. . . | |    ...
. . . |
. . . | |    ...
. . . | THEN Fold `member` 0
. . . | |
. . . | 1. n : ℕ
. . . | 2. ℕ(n-1) List
. . . | 3. u : ℕ(n-1)
. . . | 4. v : ℕ(n-1) List
. . . | 5. ∀f:(ℕn→ℤ).
. . . |    sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . . | 6. f : ℕn→ℤ
. . . | 7. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . |    =
. . . |    sum((f o (u, u+1))(x) | x < n)
. . . | 8. i : ℕn
. . . | ⊢  f((u, u+1)(compose_flips(v)(i))) ∈ ℤ
. . . |   by GenConcl (u, u+1) = g THEN Auto
. . . |
. . . .\
. . . . 1. n : ℕ
. . . . 2. ℕ(n-1) List
. . . . 3. u : ℕ(n-1)
. . . . 4. v : ℕ(n-1) List
. . . . 5. ∀f:(ℕn→ℤ).
. . . . |  sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . . . 6. f : ℕn→ℤ
. . . . 7. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . . |  =
. . . . |  sum((f o (u, u+1))(x) | x < n)
. . . . 8. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . . |  =
. . . . |  sum(f(compose_flips([u / v])(x)) | x < n)
. . . . ⊢  sum(f(compose_flips([u / v])(x)) | x < n)
. . . . |  =
. . . . |  sum(f(x) | x < n)
. . . . | by Assert sum((f o (u, u+1))(x) | x < n) = sum(f(x) | x < n)
. . . . |     THEN Auto'
. . . . |
. . . . .....assertion .....
. . . . 1. n : ℕ
. . . . 2. ℕ(n-1) List
. . . . 3. u : ℕ(n-1)
. . . . 4. v : ℕ(n-1) List
. . . . 5. ∀f:(ℕn→ℤ).
. . . . |  sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . . . 6. f : ℕn→ℤ
. . . . 7. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . . |  =
. . . . |  sum((f o (u, u+1))(x) | x < n)
. . . . 8. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . . |  =
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. . . . |  sum(f(compose_flips([u / v])(x)) | x < n)
. . . . ⊢  sum((f o (u, u+1))(x) | x < n) = sum(f(x) | x < n)
. . . . | by InstLemma ‘sum_switch‘ [n;f;u] THENA Auto
. . . . |
. . . . 1. n : ℕ
. . . . 2. ℕ(n-1) List
. . . . 3. u : ℕ(n-1)
. . . . 4. v : ℕ(n-1) List
. . . . 5. ∀f:(ℕn→ℤ).
. . . .    sum((f o compose_flips(v))(x) | x < n) = sum(f(x) | x < n)
. . . . 6. f : ℕn→ℤ
. . . . 7. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . .    =
. . . .    sum((f o (u, u+1))(x) | x < n)
. . . . 8. sum((f o (u, u+1) o compose_flips(v))(x) | x < n)
. . . .    =
. . . .    sum(f(compose_flips([u / v])(x)) | x < n)
. . . . 9. sum(f((u, u+1)(x)) | x < n) = sum(f(x) | x < n)
. . . . ⊢  sum((f o (u, u+1))(x) | x < n) = sum(f(x) | x < n)
. . . .   by ReduceSOAps (-1) THEN Reduce 0 THEN Auto
END
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