```
sublist_filter  THM

⊢  ∀T:Type, L1,L2:T List, P:(T→𝔹).
|  L2 ⊆ filter(P;L1) ⟺ L2 ⊆ L1 & (∀x∈L2.P(x))
| by InductionOnList THEN InductionOnList THEN Reduce 0
|
|\
| 1. T : Type
| 2. T List
| 3. T List
| ⊢  ∀P:(T→𝔹). nil ⊆ nil ⟺ nil ⊆ nil & (∀x∈nil.P(x))
|   by Auto THEN Easy
|
|\
| 1. T : Type
| 2. T List
| 3. T List
| 4. u : T
| 5. v : T List
| 6. ∀P:(T→𝔹). v ⊆ filter(P;nil) ⟺ v ⊆ nil & (∀x∈v.P(x))
| ⊢  ∀P:(T→𝔹). [u / v] ⊆ nil ⟺ [u / v] ⊆ nil & (∀x∈[u / v].P(x))
|   by RWO "cons_sublist_nil" 0 THEN Auto
|
|\
| 1. T : Type
| 2. T List
| 3. u : T
| 4. v : T List
| 5. ∀L2:T List, P:(T→𝔹). L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
| 6. T List
| ⊢  ∀P:(T→𝔹).
|    nil ⊆ if P(u)→ [u / filter(P;v)] else filter(P;v) fi
|    ⟺
|    nil ⊆ [u / v] & (∀x∈nil.P(x))
|   by RWO "nil_sublist" 0 THEN Auto THEN Easy
|
.\
. 1. T : Type
. 2. T List
. 3. u : T
. 4. v : T List
. 5. ∀L2:T List, P:(T→𝔹). L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. 6. T List
. 7. u1 : T
. 8. v1 : T List
. 9. ∀P:(T→𝔹). v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. ⊢  ∀P:(T→𝔹).
. |  [u1 / v1] ⊆ if P(u)→ [u / filter(P;v)] else filter(P;v) fi
. |  ⟺
. |  [u1 / v1] ⊆ [u / v] & (∀x∈[u1 / v1].P(x))
. | by D 0 THENA Auto THEN RWO "l_all_cons" 0 THENA Auto THEN
.
. |    SplitOnConclITE THENA Auto
. |
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. | .....truecase .....
. | 1. T : Type
. | 2. T List
. | 3. u : T
. | 4. v : T List
. | 5. ∀L2:T List, P:(T→𝔹). L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | 6. T List
. | 7. u1 : T
. | 8. v1 : T List
. | 9. ∀P:(T→𝔹).
. | |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | 10. P : T→𝔹
. | 11. P(u)
. | ⊢  [u1 / v1] ⊆ [u / filter(P;v)]
. | |  ⟺
. | |  [u1 / v1] ⊆ [u / v] & P(u1) & (∀x∈v1.P(x))
. | | by RWO "cons_sublist_cons" 0 THEN Auto
. | |
. | |\
. | | 1. T : Type
. | | 2. T List
. | | 3. u : T
. | | 4. v : T List
. | | 5. ∀L2:T List, P:(T→𝔹).
. | |    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | | 6. T List
. | | 7. u1 : T
. | | 8. v1 : T List
. | | 9. ∀P:(T→𝔹).
. | |    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | | 10. P : T→𝔹
. | | 11. P(u)
. | | 12. u1 = u & v1 ⊆ filter(P;v) ∨ [u1 / v1] ⊆ filter(P;v)
. | | ⊢  u1 = u & v1 ⊆ v ∨ [u1 / v1] ⊆ v
. | |   by ...
. | |
. | |\
. | | 1. T : Type
. | | 2. T List
. | | 3. u : T
. | | 4. v : T List
. | | 5. ∀L2:T List, P:(T→𝔹).
. | | |  L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | | 6. T List
. | | 7. u1 : T
. | | 8. v1 : T List
. | | 9. ∀P:(T→𝔹).
. | | |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | | 10. P : T→𝔹
. | | 11. P(u)
. | | 12. u1 = u & v1 ⊆ filter(P;v) ∨ [u1 / v1] ⊆ filter(P;v)
. | | ⊢  P(u1)  by D (-1) THEN Auto
. | | |
. | | |\
. | | | 1. T : Type
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. | | | 2. T List
. | | | 3. u : T
. | | | 4. v : T List
. | | | 5. ∀L2:T List, P:(T→𝔹).
. | | |    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | | | 6. T List
. | | | 7. u1 : T
. | | | 8. v1 : T List
. | | | 9. ∀P:(T→𝔹).
. | | |    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | | | 10. P : T→𝔹
. | | | 11. P(u)
. | | | 12. u1 = u
. | | | 13. v1 ⊆ filter(P;v)
. | | | ⊢  P(u1)  by HypSubst (-2) 0 THEN Auto
. | | |
. | | .\
. | | . 1. T : Type
. | | . 2. T List
. | | . 3. u : T
. | | . 4. v : T List
. | | . 5. ∀L2:T List, P:(T→𝔹).
. | | .    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | | . 6. T List
. | | . 7. u1 : T
. | | . 8. v1 : T List
. | | . 9. ∀P:(T→𝔹).
. | | .    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | | . 10. P : T→𝔹
. | | . 11. P(u)
. | | . 12. [u1 / v1] ⊆ filter(P;v)
. | | . ⊢  P(u1)
. | | .   by InstHyp [[u1 / v1];P] 5 THEN Auto THEN ThinTrivial
. | | .
. | | .      THEN D (-1) THEN RWO "l_all_cons" (-1) THEN Auto
. | |
. | |\
. | | 1. T : Type
. | | 2. T List
. | | 3. u : T
. | | 4. v : T List
. | | 5. ∀L2:T List, P:(T→𝔹).
. | | |  L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | | 6. T List
. | | 7. u1 : T
. | | 8. v1 : T List
. | | 9. ∀P:(T→𝔹).
. | | |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | | 10. P : T→𝔹
. | | 11. P(u)
. | | 12. u1 = u & v1 ⊆ filter(P;v) ∨ [u1 / v1] ⊆ filter(P;v)
. | | ⊢  (∀x∈v1.P(x))  by D (-1) THENA Auto
. | | |
. | | |\
. | | | 1. T : Type
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. | | | 2. T List
. | | | 3. u : T
. | | | 4. v : T List
. | | | 5. ∀L2:T List, P:(T→𝔹).
. | | |    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | | | 6. T List
. | | | 7. u1 : T
. | | | 8. v1 : T List
. | | | 9. ∀P:(T→𝔹).
. | | |    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | | | 10. P : T→𝔹
. | | | 11. P(u)
. | | | 12. u1 = u & v1 ⊆ filter(P;v)
. | | | ⊢  (∀x∈v1.P(x))
. | | |   by InstHyp [v1;P] 5 THEN Auto THEN ThinTrivial THEN Auto
. | | |
. | | .\
. | | . 1. T : Type
. | | . 2. T List
. | | . 3. u : T
. | | . 4. v : T List
. | | . 5. ∀L2:T List, P:(T→𝔹).
. | | .    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | | . 6. T List
. | | . 7. u1 : T
. | | . 8. v1 : T List
. | | . 9. ∀P:(T→𝔹).
. | | .    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | | . 10. P : T→𝔹
. | | . 11. P(u)
. | | . 12. [u1 / v1] ⊆ filter(P;v)
. | | . ⊢  (∀x∈v1.P(x))
. | | .   by InstHyp [[u1 / v1];P
. | | .      ] 5 THEN Auto THEN ThinTrivial THEN Auto
. | | .
. | | .      THEN RWO "l_all_cons" (-1) THEN Auto
. | |

. | .\
. | . 1. T : Type
. | . 2. T List
. | . 3. u : T
. | . 4. v : T List
. | . 5. ∀L2:T List, P:(T→𝔹).
. | . |  L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | . 6. T List
. | . 7. u1 : T
. | . 8. v1 : T List
. | . 9. ∀P:(T→𝔹).
. | . |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | . 10. P : T→𝔹
. | . 11. P(u)
. | . 12. u1 = u & v1 ⊆ v ∨ [u1 / v1] ⊆ v
. | . 13. P(u1)
. | . 14. (∀x∈v1.P(x))
. | . ⊢  u1 = u & v1 ⊆ filter(P;v) ∨ [u1 / v1] ⊆ filter(P;v)
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. | . | by ParallelOp (-3) THEN Auto THEN BackThruSomeHyp THEN Auto
. | . |
. | . 1. T : Type
. | . 2. T List
. | . 3. u : T
. | . 4. v : T List
. | . 5. ∀L2:T List, P:(T→𝔹).
. | .    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. | . 6. T List
. | . 7. u1 : T
. | . 8. v1 : T List
. | . 9. ∀P:(T→𝔹).
. | .    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. | . 10. P : T→𝔹
. | . 11. P(u)
. | . 12. [u1 / v1] ⊆ v
. | . 13. P(u1)
. | . 14. (∀x∈v1.P(x))
. | . ⊢  (∀x∈[u1 / v1].P(x))  by RWO "l_all_cons" 0 THEN Auto
. |
. .\
. . .....falsecase .....
. . 1. T : Type
. . 2. T List
. . 3. u : T
. . 4. v : T List
. . 5. ∀L2:T List, P:(T→𝔹). L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . 6. T List
. . 7. u1 : T
. . 8. v1 : T List
. . 9. ∀P:(T→𝔹).
. . |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . 10. P : T→𝔹
. . 11. ¬P(u)
. . ⊢  [u1 / v1] ⊆ filter(P;v)
. . |  ⟺
. . |  [u1 / v1] ⊆ [u / v] & P(u1) & (∀x∈v1.P(x))
. . | by D 0 THEN D 0 THENA Auto
. . |
. . |\
. . | 1. T : Type
. . | 2. T List
. . | 3. u : T
. . | 4. v : T List
. . | 5. ∀L2:T List, P:(T→𝔹).
. . | |  L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . | 6. T List
. . | 7. u1 : T
. . | 8. v1 : T List
. . | 9. ∀P:(T→𝔹).
. . | |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . | 10. P : T→𝔹
. . | 11. ¬P(u)
. . | 12. [u1 / v1] ⊆ filter(P;v)
. . | ⊢  [u1 / v1] ⊆ [u / v] & P(u1) & (∀x∈v1.P(x))
```

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. . | | by InstHyp [[u1 / v1];P] 5 THENA Auto
. . | |
. . | 1. T : Type
. . | 2. T List
. . | 3. u : T
. . | 4. v : T List
. . | 5. ∀L2:T List, P:(T→𝔹).
. . | |  L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . | 6. T List
. . | 7. u1 : T
. . | 8. v1 : T List
. . | 9. ∀P:(T→𝔹).
. . | |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . | 10. P : T→𝔹
. . | 11. ¬P(u)
. . | 12. [u1 / v1] ⊆ filter(P;v)
. . | 13. [u1 / v1] ⊆ filter(P;v)
. . | |   ⟺
. . | |   [u1 / v1] ⊆ v & (∀x∈[u1 / v1].P(x))
. . | ⊢  [u1 / v1] ⊆ [u / v] & P(u1) & (∀x∈v1.P(x))
. . | | by ThinTrivial THEN RWO "l_all_cons" (-1) THEN Auto
. . | |
. . | 1. T : Type
. . | 2. T List
. . | 3. u : T
. . | 4. v : T List
. . | 5. ∀L2:T List, P:(T→𝔹).
. . |    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . | 6. T List
. . | 7. u1 : T
. . | 8. v1 : T List
. . | 9. ∀P:(T→𝔹).
. . |    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . | 10. P : T→𝔹
. . | 11. ¬P(u)
. . | 12. [u1 / v1] ⊆ filter(P;v)
. . | 13. [u1 / v1] ⊆ filter(P;v) ⇐ ([u1 / v1] ⊆ v
. . |     & (∀x∈[u1 / v1].P(x)))
. . | 14. [u1 / v1] ⊆ v
. . | 15. P(u1)
. . | 16. (∀x∈v1.P(x))
. . | ⊢  [u1 / v1] ⊆ [u / v]
. . |   by RWO "cons_sublist_cons" 0 THENA Auto THEN SimpConcl
. . |
. . .\
. . . 1. T : Type
. . . 2. T List
. . . 3. u : T
. . . 4. v : T List
. . . 5. ∀L2:T List, P:(T→𝔹).
. . . |  L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . . 6. T List
. . . 7. u1 : T
. . . 8. v1 : T List
. . . 9. ∀P:(T→𝔹).
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. . . |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . . 10. P : T→𝔹
. . . 11. ¬P(u)
. . . 12. [u1 / v1] ⊆ [u / v] & P(u1) & (∀x∈v1.P(x))
. . . ⊢  [u1 / v1] ⊆ filter(P;v)  by BackThruSomeHyp THEN Auto
. . . |
. . . |\
. . . | 1. T : Type
. . . | 2. T List
. . . | 3. u : T
. . . | 4. v : T List
. . . | 5. ∀L2:T List, P:(T→𝔹).
. . . | |  L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . . | 6. T List
. . . | 7. u1 : T
. . . | 8. v1 : T List
. . . | 9. ∀P:(T→𝔹).
. . . | |  v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . . | 10. P : T→𝔹
. . . | 11. ¬P(u)
. . . | 12. [u1 / v1] ⊆ [u / v]
. . . | 13. P(u1)
. . . | 14. (∀x∈v1.P(x))
. . . | ⊢  [u1 / v1] ⊆ v
. . . | | by RWO "cons_sublist_cons" (-3) THENA Auto THEN D (-3)
. . . | |
. . . | |\
. . . | | 1. T : Type
. . . | | 2. T List
. . . | | 3. u : T
. . . | | 4. v : T List
. . . | | 5. ∀L2:T List, P:(T→𝔹).
. . . | |    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . . | | 6. T List
. . . | | 7. u1 : T
. . . | | 8. v1 : T List
. . . | | 9. ∀P:(T→𝔹).
. . . | |    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . . | | 10. P : T→𝔹
. . . | | 11. ¬P(u)
. . . | | 12. u1 = u & v1 ⊆ v
. . . | | 13. P(u1)
. . . | | 14. (∀x∈v1.P(x))
. . . | | ⊢  [u1 / v1] ⊆ v
. . . | |   by D (-3) THEN D (-5) THENA Auto THEN WeakSubstFor u
. . . | |       0 THEN Auto
. . . | |
. . . | .\
. . . | . 1. T : Type
. . . | . 2. T List
. . . | . 3. u : T
. . . | . 4. v : T List
. . . | . 5. ∀L2:T List, P:(T→𝔹).
. . . | .    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . . | . 6. T List
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. . . | . 7. u1 : T
. . . | . 8. v1 : T List
. . . | . 9. ∀P:(T→𝔹).
. . . | .    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . . | . 10. P : T→𝔹
. . . | . 11. ¬P(u)
. . . | . 12. [u1 / v1] ⊆ v
. . . | . 13. P(u1)
. . . | . 14. (∀x∈v1.P(x))
. . . | . ⊢  [u1 / v1] ⊆ v  by Trivial
. . . |
. . . .\
. . . . 1. T : Type
. . . . 2. T List
. . . . 3. u : T
. . . . 4. v : T List
. . . . 5. ∀L2:T List, P:(T→𝔹).

. . . .    L2 ⊆ filter(P;v) ⟺ L2 ⊆ v & (∀x∈L2.P(x))
. . . . 6. T List
. . . . 7. u1 : T
. . . . 8. v1 : T List
. . . . 9. ∀P:(T→𝔹).
. . . .    v1 ⊆ filter(P;[u / v]) ⟺ v1 ⊆ [u / v] & (∀x∈v1.P(x))
. . . . 10. P : T→𝔹
. . . . 11. ¬P(u)
. . . . 12. [u1 / v1] ⊆ [u / v]
. . . . 13. P(u1)
. . . . 14. (∀x∈v1.P(x))
. . . . ⊢  (∀x∈[u1 / v1].P(x))  by RWO "l_all_cons" 0 THEN Auto
END
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