double_sum_difference THM

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⊢  ∀n,m:ℕ, f,g:(ℕn→ℕm→ℤ), d:ℤ.
|  sum(f(x,y)-g(x,y) | x < n; y < m) = d
|  ⇒
|  sum(f(x,y) | x < n; y < m) = sum(g(x,y) | x < n; y < m)+d  by Auto
|
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm→ℤ
4. g : ℕn→ℕm→ℤ
5. d : ℤ
6. sum(f(x,y)-g(x,y) | x < n; y < m) = d
⊢  sum(f(x,y) | x < n; y < m) = sum(g(x,y) | x < n; y < m)+d
| by All (Unfold ‘double_sum‘)
|
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm→ℤ
4. g : ℕn→ℕm→ℤ
5. d : ℤ
6. sum(sum(f(x,y)-g(x,y) | y < m) | x < n) = d
⊢  sum(sum(f(x,y) | y < m) | x < n)
|  =
|  sum(sum(g(x,y) | y < m) | x < n)+d
| by BackThruLemma ‘sum_difference‘ THENA Auto
|
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm→ℤ
4. g : ℕn→ℕm→ℤ
5. d : ℤ
6. sum(sum(f(x,y)-g(x,y) | y < m) | x < n) = d
⊢  sum(sum(f(x,y) | y < m)-sum(g(x,y) | y < m) | x < n) = d
| by ...
|
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm→ℤ
4. g : ℕn→ℕm→ℤ
5. d : ℤ
6. sum(sum(f(x,y)-g(x,y) | y < m) | x < n) = d
7. i : ℕn
⊢  sum(f(i,y) | y < m)-sum(g(i,y) | y < m)
|  =
|  sum(f(i,y)-g(i,y) | y < m)
| by Assert sum(f(i,y) | y < m)
|           =
|           sum(g(i,y) | y < m)+sum(f(i,y)-g(i,y) | y < m) THEN Auto’
|
.....assertion .....
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm→ℤ
4. g : ℕn→ℕm→ℤ
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5. d : ℤ
6. sum(sum(f(x,y)-g(x,y) | y < m) | x < n) = d
7. i : ℕn
⊢  sum(f(i,y) | y < m)
   =
   sum(g(i,y) | y < m)+sum(f(i,y)-g(i,y) | y < m)
  by BackThruLemma ‘sum_difference‘ THEN Auto
END
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