```
fun_exp_compose  THM

⊢  ∀T:Type, n:ℕ, h,f:(T→T). f^n o h = primrec(n;h;λi,g. f o g)
| by ...
THEN Reduce 0
|
|\
| 1. T : Type
| ⊢  ∀h,f:(T→T). (λx.x) o h = h
|   by Auto THEN Ext THEN Reduce 0 THEN Auto
|
.\
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. ⊢  ∀h,f:(T→T).
. |  primrec(n;λx.x;λi,g. f o g) o h = primrec(n;h;λi,g. f o g)
. | by Auto THEN Subst' n = 1+n-1 0 THENA Auto'
. |
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. 5. h : T→T
. 6. f : T→T
. ⊢  primrec(1+n-1;λx.x;λi,g. f o g) o h
. |  =
. |  primrec(1+n-1;h;λi,g. f o g)
. | by RWO "primrec_add" 0 THEN Auto THEN Reduce 0
. |
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. 5. h : T→T
. 6. f : T→T
. ⊢  primrec(1+n-1;λx.x;λi,g. f o g) o h
. |  =
. |  f o primrec(n-1;h;λi,g. f o g)
. | by Subst primrec(1+n-1;λx.x;λi,g. f o g)
. |          =
. |          f o primrec(n-1;λx.x;λi,g. f o g) 0 THEN Auto
. |
. |\
. | .....equality .....
. | 1. T : Type
. | 2. n : ℤ
. | 3. 0<n
. | 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. | 5. T→T
. | 6. f : T→T
. | ⊢  primrec(1+n-1;λx.x;λi,g. f o g)
. | |  =
. | |  f o primrec(n-1;λx.x;λi,g. f o g)
```

```
. | | by GenConcl (λx.x) = id THENA Auto
. | |
. | 1. T : Type
. | 2. n : ℤ
. | 3. 0<n
. | 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. | 5. T→T
. | 6. f : T→T
. | 7. id : T→T
. | 8. (λx.x) = id
. | ⊢  primrec(1+n-1;id;λi,g. f o g) = f o primrec(n-1;id;λi,g. f o g)
. |   by RWO "primrec_add" 0 THEN Auto THEN Reduce 0 THEN Auto
. |
. .\
. . 1. T : Type
. . 2. n : ℤ
. . 3. 0<n
. . 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. . 5. h : T→T
. . 6. f : T→T
. . ⊢  f o primrec(n-1;λx.x;λi,g. f o g) o h
. . |  =
. . |  f o primrec(n-1;h;λi,g. f o g)
. . | by Fold `fun_exp` 0 THEN Subst f o f^n-1 o h = f o f^n-1 o h
. . |     0 THEN Auto
. . |
. . |\
. . | .....equality .....
. . | 1. T : Type
. . | 2. n : ℤ
. . | 3. 0<n
. . | 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. . | 5. h : T→T
. . | 6. f : T→T
. . | ⊢  f o f^n-1 o h = f o f^n-1 o h
. . |   by Ext THEN Reduce 0 THEN Auto
. . |
. . .\
. . . 1. T : Type
. . . 2. n : ℤ
. . . 3. 0<n
. . . 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. . . 5. h : T→T
. . . 6. f : T→T
. . . ⊢  f o f^n-1 o h = f o primrec(n-1;h;λi,g. f o g)
. . . | by Subst f^n-1 o h = primrec(n-1;h;λi,g. f o g) 0 THEN
. . . Auto
. . . |
. . . .....equality .....
. . . 1. T : Type
. . . 2. n : ℤ
. . . 3. 0<n
. . . 4. ∀h,f:(T→T). f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. . . 5. h : T→T
. . . 6. f : T→T
```

```
. . . ⊢  f^n-1 o h = primrec(n-1;h;λi,g. f o g)
. . .   by BackThruSomeHyp THEN Auto
END
```