```
increasing_inj  THM

⊢  ∀k,m:ℕ, f:(ℕk→ℕm). increasing(f;k) ⇒ Inj(ℕk; ℕm; f)
| by Unfold ‘inject‘ 0 THEN Auto
|
1. k : ℕ
2. m : ℕ
3. f : ℕk→ℕm
4. increasing(f;k)
5. a1 : ℕk
6. a2 : ℕk
7. f(a1) = f(a2)
⊢  a1 = a2  by FwdThruLemma ‘increasing_implies‘ [4] THENA Auto
|
1. k : ℕ
2. m : ℕ
3. f : ℕk→ℕm
4. increasing(f;k)
5. a1 : ℕk
6. a2 : ℕk
7. f(a1) = f(a2)
8. ∀x,y:ℕk. x<y ⇒ f(x)<f(y)
⊢  a1 = a2  by Decide a1<a2 THENA Auto
|
|\
| 1. k : ℕ
| 2. m : ℕ
| 3. f : ℕk→ℕm
| 4. increasing(f;k)
| 5. a1 : ℕk
| 6. a2 : ℕk
| 7. f(a1) = f(a2)
| 8. ∀x,y:ℕk. x<y ⇒ f(x)<f(y)
| 9. a1<a2
| ⊢  a1 = a2  by FwdThruHyp (-2) [(-1)] THEN Auto’
|
.\
. 1. k : ℕ
. 2. m : ℕ
. 3. f : ℕk→ℕm
. 4. increasing(f;k)
. 5. a1 : ℕk
. 6. a2 : ℕk
. 7. f(a1) = f(a2)
. 8. ∀x,y:ℕk. x<y ⇒ f(x)<f(y)
. 9. ¬a1<a2
. ⊢  a1 = a2  by Decide a2<a1 THENA Auto
. |
. |\
. | 1. k : ℕ
. | 2. m : ℕ
. | 3. f : ℕk→ℕm
. | 4. increasing(f;k)
. | 5. a1 : ℕk
. | 6. a2 : ℕk
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. | 7. f(a1) = f(a2)
. | 8. ∀x,y:ℕk. x<y ⇒ f(x)<f(y)
. | 9. ¬a1<a2
. | 10. a2<a1
. | ⊢  a1 = a2  by FwdThruHyp (-3) [(-1)] THEN Auto'
. |
. .\
. . 1. k : ℕ
. . 2. m : ℕ
. . 3. f : ℕk→ℕm
. . 4. increasing(f;k)
. . 5. a1 : ℕk
. . 6. a2 : ℕk
. . 7. f(a1) = f(a2)
. . 8. ∀x,y:ℕk. x<y ⇒ f(x)<f(y)
. . 9. ¬a1<a2
. . 10. ¬a2<a1
. . ⊢  a1 = a2  by Auto
END
```