```
isolate_summand  THM

⊢  ∀n:ℕ, f:(ℕn→ℤ), m:ℕn.
|  sum(f(x) | x < n) = f(m)+sum(if x=2m→ 0 else f(x) fi | x < n)
| by D 0 THENA Auto THEN NatInd 1 THEN Reduce 0
|
|\
| .....basecase .....
| ⊢  ∀f:(ℕ0→ℤ), m:ℕ0.
|    sum(f(x) | x < 0) = f(m)+sum(if x=2m→ 0 else f(x) fi | x < 0)
|   by Auto
|
.\
. .....upcase .....
. 1. n : ℤ
. 2. 0<n
. 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
. |  sum(f(x) | x < n-1) = f(m)+sum(if x=2m→ 0 else f(x) fi | x < n-1)
. ⊢  ∀f:(ℕn→ℤ), m:ℕn.
. |  sum(f(x) | x < n) = f(m)+sum(if x=2m→ 0 else f(x) fi | x < n)
. | by ParallelOp (-1) THEN Auto
. THEN Decide m = n-1 THENA Auto
. |
. |\
. | 1. n : ℤ
. | 2. 0<n
. | 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
. | |  sum(f(x) | x < n-1)
. | |  =
. | |  f(m)+sum(if x=2m→ 0 else f(x) fi | x < n-1)
. | 4. f : ℕn→ℤ
. | 5. ∀m:ℕ(n-1).
. | |  sum(f(x) | x < n-1)
. | |  =
. | |  f(m)+sum(if x=2m→ 0 else f(x) fi | x < n-1)
. | 6. m : ℕn
. | 7. m = n-1
. | ⊢  sum(f(x) | x < n) = f(m)+sum(if x=2m→ 0 else f(x) fi | x < n)
. | | by HypSubstSq (-1) 0
. | |
. | 1. n : ℤ
. | 2. 0<n
. | 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
. | |  sum(f(x) | x < n-1)
. | |  =
. | |  f(m)+sum(if x=2m→ 0 else f(x) fi | x < n-1)
. | 4. f : ℕn→ℤ
. | 5. ∀m:ℕ(n-1).
. | |  sum(f(x) | x < n-1)
. | |  =
. | |  f(m)+sum(if x=2m→ 0 else f(x) fi | x < n-1)
. | 6. m : ℕn
. | 7. m = n-1
. | ⊢  sum(f(x) | x < n)
. | |  =
```

```
. | |  f(n-1)+sum(if x=₂n-1→ 0 else f(x) fi | x < n)
. | | by Unfold ‘sum‘ 0 THEN RecUnfold ‘primrec‘ 0 THEN Fold ‘sum‘ 0
. | |
. | THEN RepeatFor 2 (SplitOnConclITE THEN
. |
. | |                      Reduce 0 THEN Auto’)
. | |
. | .....truecase .....
. | 1. n : ℤ
. | 2. 0<n
. | 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
. | |  sum(f(x) | x < n-1)
. | |  =
. | |  f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. | 4. f : ℕn→ℤ
. | 5. ∀m:ℕ(n-1).
. | |  sum(f(x) | x < n-1)
. | |  =
. | |  f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. | 6. m : ℕn
. | 7. m = n-1
. | 8. ¬n = 0
. | 9. n-1 = n-1
. | ⊢  sum(f(x) | x < n-1)+f(n-1)
. | |  =
. | |  f(n-1)+sum(if x=₂n-1→ 0 else f(x) fi | x < n-1)+0
. | | by Assert sum(f(x) | x < n-1)
. | |            =
. | |            sum(if x=₂n-1→ 0 else f(x) fi | x < n-1) THEN Auto’
. | |
. | .....assertion .....
. | 1. n : ℤ
. | 2. 0<n
. | 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
. |    sum(f(x) | x < n-1)
. |    =
. |    f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. | 4. f : ℕn→ℤ
. | 5. ∀m:ℕ(n-1).
. |    sum(f(x) | x < n-1)
. |    =
. |    f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. | 6. m : ℕn
. | 7. m = n-1
. | 8. ¬n = 0
. | 9. n-1 = n-1
. | ⊢  sum(f(x) | x < n-1) = sum(if x=₂n-1→ 0 else f(x) fi | x < n-1)
. |   by BackThruLemma ‘sum_functionality‘ THEN Auto
. |
. |      THEN SplitOnConclITE THEN Auto’
. |
. .\
. . 1. n : ℤ
. . 2. 0<n
. . 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
```

```
. . |  sum(f(x) | x < n-1)
. . |  =
. . |  f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . 4. f : ℕn→ℤ
. . 5. ∀m:ℕ(n-1).
. . |  sum(f(x) | x < n-1)
. . |  =
. . |  f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . 6. m : ℕn
. . 7. ¬m = n-1
. . ⊢  sum(f(x) | x < n) = f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n)
. . | by InstHyp [m] (-3) THENA Auto
. . |
. . 1. n : ℤ
. . 2. 0<n
. . 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
. . |  sum(f(x) | x < n-1)
. . |  =
. . |  f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . 4. f : ℕn→ℤ
. . 5. ∀m:ℕ(n-1).
. . |  sum(f(x) | x < n-1)
. . |  =
. . |  f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . 6. m : ℕn
. . 7. ¬m = n-1
. . 8. sum(f(x) | x < n-1)
. . |  =
. . |  f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . ⊢  sum(f(x) | x < n) = f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n)
. . | by Unfold `sum` 0 THEN RecUnfold `primrec` 0 THEN Fold `sum` 0
. . |
. . 1. n : ℤ
. . 2. 0<n
. . 3. ∀f:(ℕ(n-1)→ℤ), m:ℕ(n-1).
. .    sum(f(x) | x < n-1)
. .    =
. .    f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . 4. f : ℕn→ℤ
. . 5. ∀m:ℕ(n-1).
. .    sum(f(x) | x < n-1)
. .    =
. .    f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . 6. m : ℕn
. . 7. ¬m = n-1
. . 8. sum(f(x) | x < n-1)
. .    =
. .    f(m)+sum(if x=₂m→ 0 else f(x) fi | x < n-1)
. . ⊢  if n=₂0→ 0 else (λx,n. n+f(x))(n-1,sum(f(x) | x < n-1)) fi
. .    =
. .    f(m)+if n=₂0→ 0
. .         else (λx,n. n+if x=₂m→ 0 else f(x) fi)
. .              (n-1
. .              ,sum(if x=₂m→ 0 else f(x) fi | x < n-1)) fi
. .   by RepeatFor 2 (SplitOnConclITE THEN
```

```
. .
. .                                 Reduce 0 THEN Auto')
END
```