```
primrec_add  THM

⊢  ∀T:Type, n,m:ℕ, b:T, c:(ℕ(n+m)→T→T).
|  primrec(n+m;b;c) = primrec(n;primrec(m;b;c);λi,t. c(i+m,t))
| by RepeatFor 2 (D 0 THENA Auto) THEN NatInd (-1) THEN
Auto
|
|\
| 1. T : Type
| 2. m : ℕ
| 3. b : T
| 4. c : ℕ(0+m)→T→T
| ⊢  primrec(0+m;b;c) = primrec(0;primrec(m;b;c);λi,t. c(i+m,t))
| | by Reduce 0
| |
| 1. T : Type
| 2. m : ℕ
| 3. b : T
| 4. c : ℕ(0+m)→T→T
| ⊢  primrec(0+m;b;c) = primrec(m;b;c)  by Subst' 0+m = m 0 THEN Auto
|
.\
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀m:ℕ, b:T, c:(ℕ(n-1+m)→T→T).
. |  primrec(n-1+m;b;c) = primrec(n-1;primrec(m;b;c);λi,t. c(i+m,t))
. 5. m : ℕ
. 6. b : T
. 7. c : ℕ(n+m)→T→T
. ⊢  primrec(n+m;b;c) = primrec(n;primrec(m;b;c);λi,t. c(i+m,t))
. | by InstHyp [m;b;c] 4 THENA Auto
. |
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀m:ℕ, b:T, c:(ℕ(n-1+m)→T→T).
. |  primrec(n-1+m;b;c) = primrec(n-1;primrec(m;b;c);λi,t. c(i+m,t))
. 5. m : ℕ
. 6. b : T
. 7. c : ℕ(n+m)→T→T
. 8. primrec(n-1+m;b;c) = primrec(n-1;primrec(m;b;c);λi,t. c(i+m,t))
. ⊢  primrec(n+m;b;c) = primrec(n;primrec(m;b;c);λi,t. c(i+m,t))
. | by MoveToConcl (-1) THEN GenConcl primrec(m;b;c) = b' THEN Auto'
.
. |    THEN Try (ExtWith ['z'] [ℕ(n+m)→T→T] THEN Auto')
. |
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀m:ℕ, b:T, c:(ℕ(n-1+m)→T→T).
. |  primrec(n-1+m;b;c) = primrec(n-1;primrec(m;b;c);λi,t. c(i+m,t))
. 5. m : ℕ
. 6. b : T
. 7. c : ℕ(n+m)→T→T
```

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. 8. b' : T
. 9. primrec(m;b;c) = b'
. 10. primrec(n-1+m;b;c) = primrec(n-1;b';λi,t. c(i+m,t))
. ⊢  primrec(n+m;b;c) = primrec(n;b';λi,t. c(i+m,t))
. | by RecUnfold `primrec` 0 THEN Reduce 0 THEN
.
. |    Repeat (SplitOnConclITE THENA Auto
.
. |            THEN Try (Complete Auto'))
. |
. .....falsecase .....
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀m:ℕ, b:T, c:(ℕ(n-1+m)→T→T).
. |  primrec(n-1+m;b;c) = primrec(n-1;primrec(m;b;c);λi,t. c(i+m,t))
. 5. m : ℕ
. 6. b : T
. 7. c : ℕ(n+m)→T→T
. 8. b' : T
. 9. primrec(m;b;c) = b'
. 10. primrec(n-1+m;b;c) = primrec(n-1;b';λi,t. c(i+m,t))
. 11. ¬n+m = 0
. 12. ¬n = 0
. ⊢  c(n+m-1,primrec(n+m-1;b;c))
. |  =
. |  c(n-1+m,primrec(n-1;b';λi,t. c(i+m,t)))
. | by Subst' n+m-1 = n-1+m 0 THENA Auto'
. |
. 1. T : Type
. 2. n : ℤ
. 3. 0<n
. 4. ∀m:ℕ, b:T, c:(ℕ(n-1+m)→T→T).
.    primrec(n-1+m;b;c) = primrec(n-1;primrec(m;b;c);λi,t. c(i+m,t))
. 5. m : ℕ
. 6. b : T
. 7. c : ℕ(n+m)→T→T
. 8. b' : T
. 9. primrec(m;b;c) = b'
. 10. primrec(n-1+m;b;c) = primrec(n-1;b';λi,t. c(i+m,t))
. 11. ¬n+m = 0
. 12. ¬n = 0
. ⊢  c(n-1+m,primrec(n-1+m;b;c))
.    =
.    c(n-1+m,primrec(n-1;b';λi,t. c(i+m,t)))
.   by HypSubst (-3) 0 THEN Auto'
END
```