```
bool_listify  THM

⊢  ∀f:(T→𝔹). Fin(T) ⇒ (∃fL:T*. ∀t:T. f(t) ⟺ mem_f(T;t;fL))
| by RepD
|
1. T : Type
2. f : T→𝔹
3. Fin(T)
⊢  ∃fL:T*. ∀t:T. f(t) ⟺ mem_f(T;t;fL)  by Analyze -1 THEN Analyze -1
|
3. n : ℕ
4. f1 : ℕn→T
5. Bij(ℕn; T; f1)
⊢
| by Assert
|    (∀k:ℕ.
|     k≤n ⇒
|     (∃fL:T*.
|      (∀t:T. mem_f(T;t;fL) ⇒ f(t))
|      & (∀i:ℕk. f(f1(i)) ⇒ mem_f(T;f1(i);fL))))
|
|\
| .....assertion .....
| ⊢  ∀k:ℕ.
| |  k≤n ⇒
| |  (∃fL:T*.
| |   (∀t:T. mem_f(T;t;fL) ⇒ f(t))
| |   & (∀i:ℕk. f(f1(i)) ⇒ mem_f(T;f1(i);fL)))
| | by (Analyze 0) THEN (NatInd -1)
| |
| |\
| | .....basecase .....
| | ⊢  0≤n ⇒
| | |  (∃fL:T*.
| | |   (∀t:T. mem_f(T;t;fL) ⇒ f(t))
| | |   & (∀i:ℕ0. f(f1(i)) ⇒ mem_f(T;f1(i);fL)))
| | | by (Analyze 0) THEN (InstConcl [nil])
| | |
| | 6. 0≤n
| | 7. t : T
| | 8. mem_f(T;t;nil)
| | ⊢  f(t)  by Reduce -1
| |
| .\
| . .....upcase .....
| . 6. k : ℤ
| . 7. 0<k
| . 8. k-1≤n ⇒
| . |  (∃fL:T*.
| . |   (∀t:T. mem_f(T;t;fL) ⇒ f(t))
| . |   & (∀i:ℕ(k-1). f(f1(i)) ⇒ mem_f(T;f1(i);fL)))
| . ⊢  k≤n ⇒
| . |  (∃fL:T*.
| . |   (∀t:T. mem_f(T;t;fL) ⇒ f(t))
| . |   & (∀i:ℕk. f(f1(i)) ⇒ mem_f(T;f1(i);fL)))
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| . | by (Analyze 0) THEN (Analyze -2) THEN (Analyze -1)
| . |    THEN
| . |    (Analyze -1)
| . |
| . 8. k≤n
| . 9. fL : T*
| . 10. ∀t:T. mem_f(T;t;fL) ⇒ f(t)
| . 11. ∀i:ℕ(k-1). f(f1(i)) ⇒ mem_f(T;f1(i);fL)
| . ⊢  ∃fL:T*.
| . |  (∀t:T. mem_f(T;t;fL) ⇒ f(t))
| . |  & (∀i:ℕk. f(f1(i)) ⇒ mem_f(T;f1(i);fL))
| . | by Decide (f(f1(k-1)))
| . |
| . |\
| . | 12. f(f1(k-1))
| . | ⊢   by (InstConcl [f1(k-1).fL]) THEN (Reduce 0)
| . | |
| . | |\
| . | | 13. t : T
| . | | 14. f1(k-1) = t ∨ mem_f(T;t;fL)
| . | | ⊢  f(t)  by Analyze -1
| . | | |
| . | | |\
| . | | | 14. f1(k-1) = t
| . | | | ⊢   by RWH (HypC -1) 12
| . | | |
| . | | .\
| . | | . 14. mem_f(T;t;fL)
| . | | . ⊢   by BackThru 10
| . | |
| . | .\
| . | . 13. i : ℕk
| . | . 14. f(f1(i))
| . | . ⊢  f1(k-1) = f1(i) ∨ mem_f(T;f1(i);fL)  by Decide (i = k-1)
| . | . |
| . | . |\
| . | . | 15. i = k-1
| . | . | ⊢   by (Sel 1 (Analyze 0)) THEN Analyze
| . | . |
| . | . .\
| . | . . 15. ¬i = k-1
| . | . . ⊢   by (Sel 2 (Analyze 0)) THEN (BackThru 11)
| . |
| . .\
| . . 12. ¬f(f1(k-1))
| . . ⊢   by InstConcl [fL]
| . . |

| . . |\
| . . | 13. t : T
| . . | 14. mem_f(T;t;fL)
| . . | ⊢  f(t)  by BackThru 10
| . . |
| . . .\
| . . . 13. i : ℕk
| . . . 14. f(f1(i))
```

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| . . . ⊢  mem_f(T;f1(i);fL)  by Decide (i = k-1)
| . . . |
| . . . |\
| . . . | 15. i = k-1
| . . . | ⊢   by Analyze 12
| . . . | |
| . . . | 12. i : ℕk
| . . . | 13. f(f1(i))
| . . . | 14. i = k-1
| . . . | ⊢  f(f1(k-1))  by Assert (f1(i) = f1(k-1)) THENA Analyze
| . . . | |
| . . . | 15. f1(i) = f1(k-1)
| . . . | ⊢   by RWH (HypC -1) -3
| . . . |
| . . . .\
| . . . . 15. ¬i = k-1
| . . . . ⊢   by BackThru 11
|
.\
. 6. ∀k:ℕ.
. |  k≤n ⇒
. |  (∃fL:T*.
. |   (∀t:T. mem_f(T;t;fL) ⇒ f(t))
. |   & (∀i:ℕk. f(f1(i)) ⇒ mem_f(T;f1(i);fL)))
. ⊢   by (InstHyp [n] -1) THEN (Analyze -1)
. |
. 7. fL : T*
. 8. (∀t:T. mem_f(T;t;fL) ⇒ f(t))
. |  & (∀i:ℕn. f(f1(i)) ⇒ mem_f(T;f1(i);fL))
. ⊢   by InstConcl [fL]
. |
. |\
. | 8. ∀t:T. mem_f(T;t;fL) ⇒ f(t)
. | 9. ∀i:ℕn. f(f1(i)) ⇒ mem_f(T;f1(i);fL)
. | 10. t : T
. | 11. f(t)
. | ⊢  mem_f(T;t;fL)  by Analyze 5 THEN Unfold ‘surject‘ 6
. | |
. | 5. Inj(ℕn; T; f1)
. | 6. ∀b:T. ∃a:ℕn. f1(a) = b
. | 7. ∀k:ℕ.
. | |  k≤n ⇒
. | |  (∃fL:T*.
. | |   (∀t:T. mem_f(T;t;fL) ⇒ f(t))
. | |   & (∀i:ℕk. f(f1(i)) ⇒ mem_f(T;f1(i);fL)))
. | 8. fL : T*
. | 9. ∀t:T. mem_f(T;t;fL) ⇒ f(t)
. | 10. ∀i:ℕn. f(f1(i)) ⇒ mem_f(T;f1(i);fL)
. | 11. t : T
. | 12. f(t)
. | ⊢
. | | by (InstHyp [t] 6) THEN (Analyze -1) THEN (RWH (RevHypC -1) 0)
. | |
. | 13. a : ℕn
. | 14. f1(a) = t
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. | ⊢  mem_f(T;f1(a);fL)  by (RWH (RevHypC -1) -3) THEN (BackThru 10)
. |
. .\
. . 8. ∀t:T. mem_f(T;t;fL) ⇒ f(t)
. . 9. ∀i:ℕn. f(f1(i)) ⇒ mem_f(T;f1(i);fL)
. . 10. t : T
. . 11. mem_f(T;t;fL)
. . ⊢  f(t)  by BackThru 8
END
```