```
mn_23  THM

⊢  ∀n:{1...}, A:Type, L:LangOver(A), R:(A*→A*→Prop).
|  Fin(A) ⇒
|  (EquivRel x,y:A*. x R y) ⇒
|  (ℕn ∼ (x,y:A*//(x R y))) ⇒
|  (∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
|  (∃g:((x,y:A*//(x R y))→𝔹). ∀l:A*. L(l) ⟺ g(l)) ⇒
|  (∃m:ℕ. ℕm ∼ (x,y:A*//(x L-induced Equiv y))) & (∀l:A*. Dec(L(l)))
| by (Analyze 0) THEN (Analyze 0) THEN (Analyze 0)
|    THEN
|    (Assert (L ∈ A*→Prop))
|
|\
| .....assertion .....
| 1. {1...}
| 2. A : Type
| 3. L : LangOver(A)
| ⊢  L ∈ A*→Prop  by Unfold ‘languages‘ 3
|
.\
. 1. n : {1...}
. 2. A : Type
. 3. L : LangOver(A)
. 4. L ∈ A*→Prop
. ⊢  ∀R:(A*→A*→Prop).
. |  Fin(A) ⇒
. |  (EquivRel x,y:A*. x R y) ⇒
. |  (ℕn ∼ (x,y:A*//(x R y))) ⇒
. |  (∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
. |  (∃g:((x,y:A*//(x R y))→𝔹). ∀l:A*. L(l) ⟺ g(l)) ⇒
. |  (∃m:ℕ. ℕm ∼ (x,y:A*//(x L-induced Equiv y))) & (∀l:A*. Dec(L(l)))
. | by GenExRepD
. |
. |\
. | 5. R : A*→A*→Prop
. | 6. Fin(A)
. | 7. EquivRel x,y:A*. x R y
. | 8. ℕn ∼ (x,y:A*//(x R y))
. | 9. ∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))
. | 10. g : (x,y:A*//(x R y))→𝔹
. | 11. ∀l:A*. L(l) ⟺ g(l)
. | ⊢  ∃m:ℕ. ℕm ∼ (x,y:A*//(x L-induced Equiv y))
. | | by Assert (∀x,y:x,y:A*//(x R y). Dec(x Rg y))
. | |
. | |\
. | | .....decidable? .....
. | | 12. x : x,y:A*//(x R y)
. | | 13. y : x,y:A*//(x R y)
. | | ⊢  Dec(x Rg y)
. | | | by BackThru
. | | |    Thm* ∀R:(Alph*→Alph*→Prop).
. | | |         Fin(Alph) ⇒
. | | |         (EquivRel x,y:Alph*. x R y) ⇒
. | | |         Fin(x,y:Alph*//(x R y)) ⇒
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. | | |          (∀x,y,z:Alph*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
. | | |          (∀g:((x,y:Alph*//(x R y))→𝔹), x,y:x,y:Alph*//(x R y).
. | | |           Dec(x Rg y))
. | | |     {mn_23_lem_1}
. | | |
. | | ⊢  Fin(x,y:A*//(x R y))  by Witness n
. | | |
. | | ⊢  ∃f:(𝕟n→(x,y:A*//(x R y))). Bij(𝕟n; x,y:A*//(x R y); f)
. | |   by BackThru
. | |      Thm* (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)
. | |      {bij_iff_1_1_corr}
. | |
. | .\
. | . 12. ∀x,y:x,y:A*//(x R y). Dec(x Rg y)
. | . ⊢   by Thin 6
. | . |
. | . 6. EquivRel x,y:A*. x R y
. | . 7. 𝕟n ∼ (x,y:A*//(x R y))
. | . 8. ∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))
. | . 9. g : (x,y:A*//(x R y))→𝔹
. | . 10. ∀l:A*. L(l) ⟺ g(l)
. | . 11. ∀x,y:x,y:A*//(x R y). Dec(x Rg y)
. | . ⊢
. | . | by Inst
. | . |    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
. | . |    {lang_rel_equi}
. | . |    [A;L]
. | . |
. | . 12. EquivRel x,y:A*. x L-induced Equiv y
. | . ⊢
. | . | by Inst
. | . |    Thm* ∀R:(A*→A*→Prop).
. | . |          (EquivRel x,y:A*. x R y) ⇒
. | . |          (∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
. | . |          (∀g:((x,y:A*//(x R y))→𝔹).
. | . |           EquivRel u,v:x,y:A*//(x R y). u Rg v)
. | . |    {lquo_rel_equi}
. | . |    [A;R;g]
. | . |
. | . 13. EquivRel u,v:x,y:A*//(x R y). u Rg v
. | . ⊢
. | . | by FwdThru
. | . |    Thm* ∀L:LangOver(Alph), R:(Alph*→Alph*→Prop).
. | . |          (EquivRel x,y:Alph*. x R y) ⇒
. | . |          (∀x,y,z:Alph*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
. | . |          (∀g:((x,y:Alph*//(x R y))→𝔹).
. | . |           (∀l:Alph*. L(l) ⟺ g(l)) ⇒
. | . |           x,y:Alph*//(x L-induced Equiv y)
. | . |           =
. | . |           x,y:Alph*//(x Rg y))
. | . |    {mn_23_Rl_equal_Rg}
. | . |    [6;10]
. | . |
. | . 14. x,y:A*//(x L-induced Equiv y) = x,y:A*//(x Rg y)
. | . ⊢
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. | . | by FwdThru
. | . |    Thm* ∀R:(T→T→Prop).
. | . |         (EquivRel x,y:T. x R y) ⇒
. | . |         (∀Q:((x,y:T//(x R y))→(x,y:T//(x R y))→Prop).
. | . |          (EquivRel u,v:x,y:T//(x R y). u Q v) ⇒
. | . |          ((x,y:T//(x Q y)) ∼ (u,v:(x,y:T//(x R y))//(u Q v))))
. | . |    {quo_of_quo}
. | . |    [13]
. | . |
. | . 15. (x,y:A*//(x Rg y)) ∼ (u,v:(x,y:A*//(x R y))//(u Rg v))
. | . ⊢
. | . | by FwdThru
. | . |    Thm* ∀n:{1...}, A:Type, R:(A→A→Prop).
. | . |         (ℕn ∼ A) ⇒
. | . |         (EquivRel x,y:A. x R y) ⇒
. | . |         (∀x,y:A. Dec(x R y)) ⇒
. | . |         (∃m:ℕ(n+1). ℕm ∼ (x,y:A//(x R y)))
. | . |    {quotient_of_finite}
. | . |    [7;13]
. | . |
. | . 16. ∃m:ℕ(n+1). ℕm ∼ (u,v:(x,y:A*//(x R y))//(u Rg v))
. | . ⊢   by (Analyze 16) THEN (InstConcl [m])
. | . |
. | . 16. m : ℕ(n+1)
. | . 17. ℕm ∼ (u,v:(x,y:A*//(x R y))//(u Rg v))
. | . ⊢  ℕm ∼ (x,y:A*//(x L-induced Equiv y))
. | . | by FwdThru Thm* (A ∼ B) ⇒ (B ∼ A)    {one_one_corr_symm}
. | . |    [15]
. | . |
. | . 18. (u,v:(x,y:A*//(x R y))//(u Rg v)) ∼ (x,y:A*//(x Rg y))
. | . ⊢
. | . | by FwdThru
. | . |    Thm* (A ∼ B) ⇒ (B ∼ C) ⇒ (A ∼ C)    {one_one_corr_tran}
. | . |    [17;18]
. | . |
. | . 19. ℕm ∼ (x,y:A*//(x Rg y))
. | . ⊢   by RevHypSubst 14 19
. |
. .\
. . 5. R : A*→A*→Prop
. . 6. Fin(A)
. . 7. EquivRel x,y:A*. x R y
. . 8. ℕn ∼ (x,y:A*//(x R y))
. . 9. ∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))
. . 10. g : (x,y:A*//(x R y))→𝔹
. . 11. ∀l:A*. L(l) ⟺ g(l)
. . 12. l : A*
. . ⊢  Dec(L(l))  by RWO "11" 0
END
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