```
mn_23_Rl_equal_Rg  THM

⊢  ∀L:LangOver(Alph), R:(Alph*→Alph*→Prop).
|  (EquivRel x,y:Alph*. x R y) ⇒
|  (∀x,y,z:Alph*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
|  (∀g:((x,y:Alph*//(x R y))→𝔹).
|   (∀l:Alph*. L(l) ⟺ g(l)) ⇒
|   x,y:Alph*//(x L-induced Equiv y) = x,y:Alph*//(x Rg y))
| by (Unfold `languages` 0) THEN UnivCD
|
1. Alph : Type
2. L : Alph*→Prop
3. R : Alph*→Alph*→Prop
4. EquivRel x,y:Alph*. x R y
5. ∀x,y,z:Alph*. (x R y) ⇒ ((z @ x) R (z @ y))
6. g : (x,y:Alph*//(x R y))→𝔹
7. ∀l:Alph*. L(l) ⟺ g(l)
⊢  x,y:Alph*//(x L-induced Equiv y) = x,y:Alph*//(x Rg y)
| by QuotEqCD
|
|\
| 8. Alph*
| 9. Alph*
| ⊢  L ∈ LangOver(Alph)  by Unfold `languages` 0
|
|\
| ⊢  EquivRel x,y:Alph*. x L-induced Equiv y
| | by BackThru
| |    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
| |    {lang_rel_equi}
| |
| .....wf .....
| ⊢  L ∈ LangOver(Alph)  by Unfold `languages` 0
|
|\
| ⊢  EquivRel x,y:Alph*. x Rg y
| | by BackThru
| |    Thm* ∀R:(T→T→Prop).
| |          (EquivRel x,y:T. x R y) ⇒
| |          (∀Q:((x,y:T//(x R y))→(x,y:T//(x R y))→Prop).
| |           (EquivRel u,v:x,y:T//(x R y). u Q v) ⇒
| |           (EquivRel x,y:T. x Q y))
| |    {incl_aux6_quo}
| |
| |\
| | ⊢  EquivRel x,y:Alph*. x R y  by Auto
| |
| .\
| . ⊢  EquivRel u,v:x,y:Alph*//(x R y). u Rg v
| . | by BackThru
| . |    Thm* ∀R:(A*→A*→Prop).
| . |          (EquivRel x,y:A*. x R y) ⇒
| . |          (∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
| . |          (∀g:((x,y:A*//(x R y))→𝔹).
| . |           EquivRel u,v:x,y:A*//(x R y). u Rg v)
```

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| . |    {lquo_rel_equi}
| . |
| . |\
| . | ⊢  EquivRel x,y:Alph*. x R y  by Auto
| . |
| . .\
| . . ⊢  ∀x,y,z:Alph*. (x R y) ⇒ ((z @ x) R (z @ y))  by Auto
|
|\
| ⊢  Alph* = Alph*  by Auto
|
.\
. 8. Alph* = Alph*
. 9. x : Alph*
. 10. y : Alph*
. ⊢  (x L-induced Equiv y) ⟺ (x Rg y)
. | by Inst
. |    Thm* ∀R:(A*→A*→Prop).
. |          (EquivRel x,y:A*. x R y) ⇒
. |          (∀x,y,z:A*. (x R y) ⇒ ((z @ x) R (z @ y))) ⇒
. |          (∀g:((x,y:A*//(x R y))→𝔹), L:LangOver(A).
. |           (∀l:A*. L(l) ⟺ g(l)) ⇒
. |           (∀x,y:A*. (x L-induced Equiv y) ⟺ (x Rg y)))
. |    {Rl_iff_Rg}
. |    [Alph;R;g;L;x;y]
. |
. .....wf .....
. ⊢  L ∈ LangOver(Alph)  by Unfold `languages` 0
END
```