```
mn_23_lem_0  THM

⊢  ∀P:(ℕ→Prop), n:ℕ.
|  (∀k:{n...}. P(k) ⇒ (∃i:ℕk. P(i))) & (∃m:ℕ. P(m)) ⇒ (∃m:ℕn. P(m))
| by UnivCD THEN Analyze 3 THEN Analyze 4
|
1. P : ℕ→Prop
2. n : ℕ
3. ∀k:{n...}. P(k) ⇒ (∃i:ℕk. P(i))
4. m : ℕ
5. P(m)
⊢  ∃m:ℕn. P(m)  by Decide (m<n)
|
|\
| 6. m<n
| ⊢   by Witness m
|
.\
. 6. ¬m<n
. ⊢   by Assert (∀i:ℕ(m-n+2). ∃l:ℕ(m-i+1). P(l))
. |
. |\
. | .....assertion .....
. | ⊢  ∀i:ℕ(m-n+2). ∃l:ℕ(m-i+1). P(l)  by Analyze 0
. | |
. | 7. i : ℕ(m-n+2)
. | ⊢  ∃l:ℕ(m-i+1). P(l)  by NSubsetInd 7
. | |
. | |\
. | | .....basecase .....
. | | 6. ¬m<n
. | | ⊢  ∃l:ℕ(m-0+1). P(l)  by Witness m
. | |
. | .\
. | . .....upcase .....
. | . 7. i : ℤ
. | . 8. 0<i
. | . 9. i<m-n+2
. | . 10. ∃l:ℕ(m-(i-1)+1). P(l)
. | . ⊢   by Analyze 10
. | . |
. | . 10. l : ℕ(m-(i-1)+1)
. | . 11. P(l)
. | . ⊢   by Decide (l<m-i+1)
. | . |
. | . |\
. | . | 12. l<m-i+1
. | . | ⊢   by Witness l
. | . |
. | . .\
. | . . 12. ¬l<m-i+1
. | . . ⊢   by Witness3 l
. | . . |
. | . . 3. m : ℕ
. | . . 4. P(m)
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. | . . 5. ¬m<n
. | . . 6. i : ℤ
. | . . 7. 0<i
. | . . 8. i<m-n+2
. | . . 9. l : ℕ(m-(i-1)+1)
. | . . 10. P(l)
. | . . 11. ¬l<m-i+1
. | . . 12. P(l) ⇒ (∃i:ℕl. P(i))
. | . . ⊢   by Analyze 12
. | . . |
. | . . 12. ∃i:ℕl. P(i)
. | . . ⊢   by Analyze 12
. | . . |
. | . . 12. i1 : ℕl
. | . . 13. P(i1)
. | . . ⊢   by Witness i1
. |
. .\
. . 7. ∀i:ℕ(m-n+2). ∃l:ℕ(m-i+1). P(l)
. . ⊢   by Witness7 m-n+1
. . |
. . 7. ∃l:ℕ(m-(m-n+1)+1). P(l)
. . ⊢   by Analyze 7
. . |
. . 7. l : ℕ(m-(m-n+1)+1)
. . 8. P(l)
. . ⊢   by Witness l
END
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