```
hd_reverse  THM

⊢  ∀T:Type, l:T*. ||l||>0 ⇒ hd(rev(l)) = l[(||l||-1)]  by UnivCD
|
1. T : Type
2. l : T*
3. ||l||>0
⊢  hd(rev(l)) = l[(||l||-1)]  by ListInd 2
|
|\
| 2. T*
| ⊢  ||nil||>0 ⇒ hd(rev(nil)) = nil[(||nil||-1)] ∈ T  by Reduce 0
|
.\
. 3. u : T
. 4. v : T*
. 5. ||v||>0 ⇒ hd(rev(v)) = v[(||v||-1)]
. ⊢  ||u.v||>0 ⇒ hd(rev(u.v)) = (u.v)[(||u.v||-1)]
. | by Decide (||v|| = 0)
. |
. |\
. | 6. ||v|| = 0
. | ⊢   by Reduce 0
. | |
. | ⊢  ||v||+1>0 ⇒ hd((rev(v) @ [u])) = (u.v)[(||v||+1-1)]
. | | by RWH
. | |    (LemmaC
. | |     Thm* ∀a:T, as:T*, i:ℤ. i≤0 ⇒ (a.as)[i] = a
. | |     {select_cons_hd})
. | |    0
. | |
. | |\
. | | 7. ||v||+1>0
. | | ⊢  ||rev(v) @ [u]||≥1
. | |   by RWH
. | |      (LemmaC
. | |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | |       {length_append})
. | |      0
. | |      THEN
. | |      Reduce 0
. | |
. | .\
. | . ⊢  ||v||+1>0 ⇒ hd((rev(v) @ [u])) = u
. | . | by RWH
. | . |    (LemmaC
. | . |     Thm* ∀l,m:T*. ||l|| = 0 ⇒ ||m||>0 ⇒ hd((l @ m)) = hd(m)
. | . |     {hd_append_back})
. | . |    0
. | . |
. | . |\
. | . | .....rewrite subgoal .....
. | . | 7. ||v||+1>0
. | . | ⊢  ||rev(v)|| = 0  by Auto
. | . |
```

```
. | . |\
. | . | .....rewrite subgoal .....
. | . | 7. ||v||+1>0
. | . | ⊢  ||[u]||>0  by Reduce 0
. | . |
. | . .\
. | . . ⊢  ||v||+1>0 ⇒ hd([u]) = u  by Reduce 0
. |
. .\
. . 6. ¬||v|| = 0
. . ⊢
. . | by RWH
. . |    (LemmaC
. . |     Thm* ∀a:T, as:T*, i:ℤ.
. . |           0<i ⇒ i≤||as|| ⇒ (a.as)[i] = as[(i-1)]
. . |     {select_cons_tl})
. . |    0
. . |
. . ⊢  ||u.v||>0 ⇒ hd(rev(u.v)) = v[(||u.v||-1-1)]  by Reduce 0
. . |
. . ⊢  ||v||+1>0 ⇒ hd((rev(v) @ [u])) = v[(||v||+1-1-1)]
. . | by RWH
. . |    (LemmaC
. . |     Thm* ∀l,m:T*. ||l||>0 ⇒ hd((l @ m)) = hd(l)
. . |     {hd_append_front})
. . |    0
. . |
. . ⊢  ||v||+1>0 ⇒ hd(rev(v)) = v[(||v||+1-1-1)]  by Analyze 5
. . |
. . 5. ¬||v|| = 0
. . 6. hd(rev(v)) = v[(||v||-1)]
. . ⊢   by RWH (HypC 6) 0
. . |
. . ⊢  ||v||+1>0 ⇒ v[(||v||-1)] = v[(||v||+1-1-1)]  by Analyze 0
. . |
. . 7. ||v||+1>0
. . ⊢  v[(||v||-1)] = v[(||v||+1-1-1)]  by Analyze
END
```