```
nd_ext_valcom  THM

⊢  ∀Alph,St:Type, NDA:NDA(Alph;St), C:NComp(Alph;St), q:St, a:Alph
|  , p:St. NDA(C)⊢ q ⇒ δNDA(q,a,p) ⇒ NDA(C+[a;p])⊢ p  by UnivCD
|
1. Alph : Type
2. St : Type
3. NDA : NDA(Alph;St)
4. C : NComp(Alph;St)
5. q : St
6. a : Alph
7. p : St
8. NDA(C)⊢ q
9. δNDA(q,a,p)
⊢  NDA(C+[a;p])⊢ p
| by Unfold ‘nd_valcom‘ 0 THEN Unfold ‘nd_comp_extend‘ 0
|
⊢  I(NDA) = 1of(hd((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])))
|  & (∀i:ℕ(||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||-1).
|      δNDA
|      (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
|      ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])))
|      ,1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)]))
|      & 2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)])
|        =
|        rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
|                       @ [<p,nil>])[i]))))
|        ∈ Alph*)
|  & 1of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]))) = p
|  & 2of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])))
|    =
|    nil
|    ∈ Alph*  by Analyze 0
|
|\
| ⊢  I(NDA) = 1of(hd((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])))
| | by Analyze 4 THEN Analyze 4
| |
| 4. C : (St×Alph*)*
| 5. ||C||>0
| 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
| 7. q : St
| 8. a : Alph
| 9. p : St
| 10. NDA(C)⊢ q
| 11. δNDA(q,a,p)
| ⊢
| | by RWH
| |    (LemmaC
| |     Thm* ∀l,m:T*. ||l||>0 ⇒ hd((l @ m)) = hd(l)
| |     {hd_append_front})
| |    0
| |
| ⊢  I(NDA) = 1of(hd(map(λc.<1of(c),a.2of(c)>;C)))
| | by RWH (LemmaC Thm* ∀l:T*. ||l||>0 ⇒ hd(l) = l[0]    {hd_select})
```

```
| |    0
| |
| ⊢  I(NDA) = 1of(map(λc.<1of(c),a.2of(c)>;C)[0])
| | by RWH
| |    (LemmaC
| |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||. map(f;as)[n] = f(as[n])
| |     {map_select})
| |    0
| |
| ⊢  I(NDA) = 1of((λc.<1of(c),a.2of(c)>)(C[0]))  by Reduce 0
| |
| ⊢  I(NDA) = 1of(C[0])  by Analyze 10
| |
| 10. I(NDA) = 1of(hd(C))
| 11. (∀i:ℕ(||C||-1).
|      δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
|      & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i])))))
|     & 1of(hd(rev(C))) = q
|     & 2of(hd(rev(C))) = nil
| 12. δNDA(q,a,p)
| ⊢
|   by RWH (LemmaC Thm* ∀l:T*. ||l||>0 ⇒ hd(l) = l[0]    {hd_select})
|      10
|
.\
. ⊢  (∀i:ℕ(||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||-1).
. |   δNDA
. |   (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. |   ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])))
. |   ,1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)]))
. |   & 2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)])
. |     =
. |     rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. |                    @ [<p,nil>])[i]))))
. |     ∈ Alph*)
. |  & 1of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]))) = p
. |  & 2of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])))
. |    =
. |    nil
. |    ∈ Alph*  by Analyze 0
. |
. |\
. | ⊢  ∀i:ℕ(||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||-1).
. | |  δNDA
. | |  (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. | |  ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])))
. | |  ,1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)]))
. | |  & 2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)])
. | |    =
. | |    rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | |                   @ [<p,nil>])[i]))))
. | | by Analyze 4 THEN Analyze 4
. | |
. | 4. C : (St×Alph*)*
. | 5. ||C||>0  [not for witness]
```

```
. | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0  [not for witness]
. | 7. q : St
. | 8. a : Alph
. | 9. p : St
. | 10. NDA(C)⊢ q
. | 11. δNDA(q,a,p)
. | ⊢   by Analyze 0
. | |
. | 12. i : ℕ(||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||-1)
. | ⊢  δNDA
. | |  (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. | |  ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])))
. | |  ,1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)]))
. | |  & 2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)])
. | |    =
. | |    rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | |                   @ [<p,nil>])[i]))))  by Analyze 12
. | |
. | 12. i : ℤ
. | 13. 0 ≤ i < ||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||-1
. | ⊢   by Analyze 13
. | |
. | 13. 0≤i
. | 14. i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||-1
. | ⊢
. | | by RWH
. | |    (LemmaC
. | |     Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | |     {length_append})
. | |    14
. | |    THEN
. | |    Reduce 14
. | |
. | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | ⊢   by Analyze 0
. | |
. | |\
. | | ⊢  δNDA
. | | |  (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. | | |  ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])))
. | | |  ,1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)]))
. | | | by Decide (i = ||C||-1)
. | | |
. | | |\
. | | | 15. i = ||C||-1
. | | | ⊢
. | | | | by RW
. | | | |    (NthC 3
. | | | |     (LemmaC
. | | | |      Thm* ∀as,bs:T*, i:{||as||..(||as||+||bs||)⁻}.
. | | | |           (as @ bs)[i] = bs[(i-||as||)]
. | | | |      {select_append_back}))
. | | | |    0
. | | | |    THEN
. | | | |    Reduce 0
```

```
. | | | |
. | | | |\
. | | | | 5. ||C||>0
. | | | | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | | 7. q : St
. | | | | 8. a : Alph
. | | | | 9. p : St
. | | | | 10. NDA(C)⊢ q
. | | | | 11. δNDA(q,a,p)
. | | | | 12. i : ℤ
. | | | | 13. 0≤i
. | | | | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | | 15. i = ||C||-1
. | | | | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | | |   by RWH
. | | | |      (LemmaC
. | | | |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | | |       {length_append})
. | | | |      0
. | | | |      THEN
. | | | |      Reduce 0
. | | | |
. | | | |\
. | | | | 5. ||C||>0
. | | | | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | | 7. q : St
. | | | | 8. a : Alph
. | | | | 9. p : St
. | | | | 10. NDA(C)⊢ q
. | | | | 11. δNDA(q,a,p)
. | | | | 12. i : ℤ
. | | | | 13. 0≤i
. | | | | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | | 15. i = ||C||-1
. | | | | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | | |   by RWH
. | | | |      (LemmaC
. | | | |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | | |       {length_append})
. | | | |      0
. | | | |      THEN
. | | | |      Reduce 0
. | | | |
. | | | |\
. | | | | 5. ||C||>0
. | | | | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | | 7. q : St
. | | | | 8. a : Alph
. | | | | 9. p : St
. | | | | 10. NDA(C)⊢ q
. | | | | 11. δNDA(q,a,p)
. | | | | 12. i : ℤ
. | | | | 13. 0≤i
. | | | | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | | 15. i = ||C||-1
```

```
. | | | | | ⊢  ||rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | | |             @ [<p,nil>])[i]))||≥1
. | | | | | by RWH
. | | | | |    (LemmaC
. | | | | |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ
. | | | | |     {length_reverse})
. | | | | |    0
. | | | | |
. | | | | |\
. | | | | | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | | | |   by RWH
. | | | | |      (LemmaC
. | | | | |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | | | |       {length_append})
. | | | | |      0
. | | | | |      THEN
. | | | | |      Reduce 0
. | | | | |
. | | | | .\
. | | | | . ⊢  ||2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])||≥1
. | | | | . | by RWH
. | | | | . |    (LemmaC
. | | | | . |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | | | | . |     {select_append_front})
. | | | | . |    0
. | | | | . |
. | | | | . ⊢  ||2of(map(λc.<1of(c),a.2of(c)>;C)[i])||≥1
. | | | | .   by RWH
. | | | | .      (LemmaC
. | | | | .       Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | | | .            map(f;as)[n] = f(as[n])
. | | | | .       {map_select})
. | | | | .      0
. | | | | .      THEN
. | | | | .      Reduce 0
. | | | |
. | | | |\
. | | | | 5. ||C||>0
. | | | | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | | 7. q : St
. | | | | 8. a : Alph
. | | | | 9. p : St
. | | | | 10. NDA(C)⊢ q
. | | | | 11. δNDA(q,a,p)
. | | | | 12. i : ℤ
. | | | | 13. 0≤i
. | | | | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | | 15. i = ||C||-1
. | | | | ⊢  ||map(λc.<1of(c),a.2of(c)>;C)||≤i+1  by Auto
. | | | |
. | | | .\
. | | | . ⊢  δNDA
. | | | . |  (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. | | | . |  ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . |             @ [<p,nil>])[i])))
```

```
. | | | . |  ,1of([<p,nil>][(i+1-||map(λc.<1of(c),a.2of(c)>;C)||)]))
. | | | . | by Assert (i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0)
. | | | . |
. | | | . 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . ⊢   by HypSubst 16 0
. | | | . |
. | | | . |\
. | | | . | ⊢  δNDA
. | | | . | |  (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. | | | . | |  ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . | |              @ [<p,nil>])[i])))
. | | | . | |  ,1of([<p,nil>][0]))
. | | | . | | by RW (NthC 3 (UnfoldTopC ‘select‘)) 0
. | | | . | |
. | | | . | ⊢  δNDA
. | | | . | |  (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. | | | . | |  ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . | |              @ [<p,nil>])[i])))
. | | | . | |  ,1of(hd(nth_tl(0;[<p,nil>]))))
. | | | . | | by RecUnfold ‘nth_tl‘ 0 THEN Reduce 0
. | | | . | |
. | | | . | ⊢  δNDA
. | | | . | |  (1of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])
. | | | . | |  ,hd(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . | |              @ [<p,nil>])[i])))
. | | | . | |  ,p)
. | | | . | | by RWH
. | | | . | |    (LemmaC
. | | | . | |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | | | . | |     {select_append_front})
. | | | . | |    0
. | | | . | |
. | | | . | |\
. | | | . | | .....antecedent .....
. | | | . | | 5. ||C||>0
. | | | . | | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . | | 7. q : St
. | | | . | | 8. a : Alph
. | | | . | | 9. p : St
. | | | . | | 10. NDA(C)⊢ q
. | | | . | | 11. δNDA(q,a,p)
. | | | . | | 12. i : ℤ
. | | | . | | 13. 0≤i
. | | | . | | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | | 15. i = ||C||-1
. | | | . | | 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | | ⊢  ||rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . | | |           @ [<p,nil>])[i]))||≥1
. | | | . | | | by RWH
. | | | . | | |    (LemmaC
. | | | . | | |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ
. | | | . | | |     {length_reverse})
. | | | . | | |    0
. | | | . | | |
. | | | . | | |\
```

```
. | | | . | | | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | | . | | |   by RWH
. | | | . | | |      (LemmaC
. | | | . | | |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | | . | | |       {length_append})
. | | | . | | |      0
. | | | . | | |      THEN
. | | | . | | |      Reduce 0
. | | | . | | |
. | | | . | | .\
. | | | . | | . ⊢  ||2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . | | . |        @ [<p,nil>])[i])||≥1
. | | | . | | . | by RWH
. | | | . | | . |    (LemmaC
. | | | . | | . |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | | | . | | . |     {select_append_front})
. | | | . | | . |    0
. | | | . | | . |
. | | | . | | . ⊢  ||2of(map(λc.<1of(c),a.2of(c)>;C)[i])||≥1
. | | | . | | . | by RWH
. | | | . | | . |    (LemmaC
. | | | . | | . |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | | . | | . |          map(f;as)[n] = f(as[n])
. | | | . | | . |     {map_select})
. | | | . | | . |    0
. | | | . | | . |
. | | | . | | . ⊢  ||2of((λc.<1of(c),a.2of(c)>)(C[i]))||≥1
. | | | . | | .   by Reduce 0
. | | | . | |
. | | | . | .\
. | | | . | . ⊢  δNDA
. | | | . | . |  (1of(map(λc.<1of(c),a.2of(c)>;C)[i])
. | | | . | . |  ,hd(rev(2of(map(λc.<1of(c),a.2of(c)>;C)[i])))
. | | | . | . |  ,p)
. | | | . | . | by RWH
. | | | . | . |    (LemmaC
. | | | . | . |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | | . | . |          map(f;as)[n] = f(as[n])
. | | | . | . |     {map_select})
. | | | . | . |    0
. | | | . | . |    THEN
. | | | . | . |    Reduce 0
. | | | . | . |
. | | | . | . |\
. | | | . | . | .....antecedent .....
. | | | . | . | 5. ||C||>0
. | | | . | . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . | . | 7. q : St
. | | | . | . | 8. a : Alph
. | | | . | . | 9. p : St
. | | | . | . | 10. NDA(C)⊢ q
. | | | . | . | 11. δNDA(q,a,p)
. | | | . | . | 12. i : ℤ
. | | | . | . | 13. 0≤i
. | | | . | . | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
```

```
. | | | . | . | 15. i = ||C||-1
. | | | . | . | 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . | ⊢  ||rev(2of(map(λc.<1of(c),a.2of(c)>;C)[i]))||≥1
. | | | . | . | | by RWH
. | | | . | . | |    (LemmaC
. | | | . | . | |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ
. | | | . | . | |     {length_reverse})
. | | | . | . | |    0
. | | | . | . | |
. | | | . | . | ⊢  ||2of(map(λc.<1of(c),a.2of(c)>;C)[i])||≥1
. | | | . | . |   by RWH
. | | | . | . |      (LemmaC
. | | | . | . |       Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | | . | . |            map(f;as)[n] = f(as[n])
. | | | . | . |       {map_select})
. | | | . | . |      0
. | | | . | . |      THEN
. | | | . | . |      Reduce 0
. | | | . | . |
. | | | . | . .\
. | | | . | . . ⊢  δNDA(1of(C[i]),hd((rev(2of(C[i])) @ [a])),p)
. | | | . | . . | by Analyze 10
. | | | . | . . |
. | | | . | . . 10. I(NDA) = 1of(hd(C))
. | | | . | . . 11. (∀i:ℕ(||C||-1).
. | | | . | . . |     δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | | . | . . |     & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i])))))
. | | | . | . . |    & 1of(hd(rev(C))) = q
. | | | . | . . |    & 2of(hd(rev(C))) = nil
. | | | . | . . 12. δNDA(q,a,p)
. | | | . | . . 13. i : ℤ
. | | | . | . . 14. 0≤i
. | | | . | . . 15. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | . . 16. i = ||C||-1
. | | | . | . . 17. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . ⊢   by Analyze 11
. | | | . | . . |
. | | | . | . . 11. ∀i:ℕ(||C||-1).
. | | | . | . . |    δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | | . | . . |    & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | | . | . . 12. 1of(hd(rev(C))) = q & 2of(hd(rev(C))) = nil
. | | | . | . . 13. δNDA(q,a,p)
. | | | . | . . 14. i : ℤ
. | | | . | . . 15. 0≤i
. | | | . | . . 16. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | . . 17. i = ||C||-1
. | | | . | . . 18. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . ⊢   by Analyze 12
. | | | . | . . |
. | | | . | . . 12. 1of(hd(rev(C))) = q
. | | | . | . . 13. 2of(hd(rev(C))) = nil
. | | | . | . . 14. δNDA(q,a,p)
. | | | . | . . 15. i : ℤ
. | | | . | . . 16. 0≤i
. | | | . | . . 17. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
```

```
. | | | . | . . 18. i = ||C||-1
. | | | . | . . 19. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . ⊢   by Assert (rev(2of(C[i])) = nil)
. | | | . | . . |
. | | | . | . . |\
. | | | . | . . | .....assertion .....
. | | | . | . . | 5. ||C||>0
. | | | . | . . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . | . . | 7. q : St
. | | | . | . . | 8. a : Alph
. | | | . | . . | 9. p : St
. | | | . | . . | 10. I(NDA) = 1of(hd(C))
. | | | . | . . | 11. ∀i:ℕ(||C||-1).
. | | | . | . . | |   δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | | . | . . | |   & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | | . | . . | 12. 1of(hd(rev(C))) = q
. | | | . | . . | 13. 2of(hd(rev(C))) = nil
. | | | . | . . | 14. δNDA(q,a,p)
. | | | . | . . | 15. i : ℤ
. | | | . | . . | 16. 0≤i
. | | | . | . . | 17. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | . . | 18. i = ||C||-1
. | | | . | . . | 19. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . | ⊢  rev(2of(C[i])) = nil
. | | | . | . . | | by RWH
. | | | . | . . | |    (LemmaC
. | | | . | . . | |     Thm* ∀l:T*.
. | | | . | . . | |          ||l||>0 ⇒ hd(rev(l)) = l[(||l||-1)]
. | | | . | . . | |     {hd_reverse})
. | | | . | . . | |    13
. | | | . | . . | |
. | | | . | . . | 13. 2of(C[(||C||-1)]) = nil
. | | | . | . . | 14. δNDA(q,a,p)
. | | | . | . . | 15. i : ℤ
. | | | . | . . | 16. 0≤i
. | | | . | . . | 17. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | . . | 18. i = ||C||-1
. | | | . | . . | 19. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . | ⊢   by RWH (RevHypC 18) 13
. | | | . | . . | |
. | | | . | . . | 13. δNDA(q,a,p)
. | | | . | . . | 14. i : ℤ
. | | | . | . . | 15. 0≤i
. | | | . | . . | 16. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | . . | 17. i = ||C||-1
. | | | . | . . | 18. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . | 19. 2of(C[i]) = nil
. | | | . | . . | ⊢   by ApFunToHypEquands ‘x’ rev(x) (Alph*) 19
. | | | . | . . | |
. | | | . | . . | 20. rev(2of(C[i])) = rev(nil)
. | | | . | . . | ⊢   by RW (NthC 2 (RecUnfoldTopC ‘reverse‘)) 20
. | | | . | . . | |
. | | | . | . . | 20. rev(2of(C[i]))
. | | | . | . . | |   =
. | | | . | . . | |   (Case of nil
```

```
. | | | . | . . | |      nil → nil
. | | | . | . . | |      a.as' → rev(as') @ [a])
. | | | . | . . | ⊢   by ApFunToHypEquands 'x' rev(x) (Alph*) 19
. | | | . | . . | |
. | | | . | . . | 21. rev(2of(C[i])) = rev(nil)
. | | | . | . . | ⊢
. | | | . | . . |   by RW (NthC 2 (RecUnfoldTopC 'reverse')) 21
. | | | . | . . |      THEN
. | | | . | . . |      Reduce 21
. | | | . | . . |
. | | | . | . . .\
. | | | . | . . . 20. rev(2of(C[i])) = nil
. | | | . | . . . ⊢
. | | | . | . . . | by RWH
. | | | . | . . . |    (LemmaC
. | | | . | . . . |     Thm* ∀l,m:T*.
. | | | . | . . . |          ||l|| = 0 ⇒
. | | | . | . . . |          ||m||>0 ⇒ hd((l @ m)) = hd(m)
. | | | . | . . . |     {hd_append_back})
. | | | . | . . . |    0
. | | | . | . . . |    THENA
. | | | . | . . . |    (Auto THEN Reduce 0)
. | | | . | . . . |
. | | | . | . . . |\
. | | | . | . . . | .....rewrite subgoal .....
. | | | . | . . . | 5. ||C||>0
. | | | . | . . . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . | . . . | 7. q : St
. | | | . | . . . | 8. a : Alph
. | | | . | . . . | 9. p : St
. | | | . | . . . | 10. I(NDA) = 1of(hd(C))
. | | | . | . . . | 11. ∀i:ℕ(||C||-1).
. | | | . | . . . | |   δNDA
. | | | . | . . . | |   (1of(C[i])
. | | | . | . . . | |   ,hd(rev(2of(C[i])))
. | | | . | . . . | |   ,1of(C[(i+1)]))
. | | | . | . . . | |   & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | | . | . . . | 12. 1of(hd(rev(C))) = q
. | | | . | . . . | 13. 2of(hd(rev(C))) = nil
. | | | . | . . . | 14. δNDA(q,a,p)
. | | | . | . . . | 15. i : ℤ
. | | | . | . . . | 16. 0≤i
. | | | . | . . . | 17. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | . . . | 18. i = ||C||-1
. | | | . | . . . | 19. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . . | 20. rev(2of(C[i])) = nil
. | | | . | . . . | ⊢  ||rev(2of(C[i]))|| = 0  by RWH (HypC 20) 0
. | | | . | . . . | |
. | | | . | . . . | ⊢  ||nil|| = 0  by Reduce 0
. | | | . | . . . |
. | | | . | . . . .\
. | | | . | . . . . ⊢  δNDA(1of(C[i]),hd([a]),p)  by Reduce 0
. | | | . | . . . . |
. | | | . | . . . . ⊢  δNDA(1of(C[i]),a,p)
. | | | . | . . . . | by RWH
```

```
. | | | . | . . . . |    (LemmaC
. | | | . | . . . . |     Thm* ∀l:T*.
. | | | . | . . . . |          ||l||>0 ⇒ hd(rev(l)) = l[(||l||-1)]
. | | | . | . . . . |     {hd_reverse})
. | | | . | . . . . |    12
. | | | . | . . . . |
. | | | . | . . . . 12. 1of(C[(||C||-1)]) = q
. | | | . | . . . . 13. 2of(hd(rev(C))) = nil
. | | | . | . . . . 14. δNDA(q,a,p)
. | | | . | . . . . 15. i : ℤ
. | | | . | . . . . 16. 0≤i
. | | | . | . . . . 17. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | . . . . 18. i = ||C||-1
. | | | . | . . . . 19. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | . . . . 20. rev(2of(C[i])) = nil
. | | | . | . . . . ⊢   by RWH (HypC 18) 0
. | | | . | . . . . |
. | | | . | . . . . ⊢  δNDA(1of(C[(||C||-1)]),a,p)
. | | | . | . . . .   by RWH (HypC 12) 0
. | | | . |
. | | | . |\
. | | | . | 5. ||C||>0
. | | | . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . | 7. q : St
. | | | . | 8. a : Alph
. | | | . | 9. p : St
. | | | . | 10. NDA(C)⊢ q
. | | | . | 11. δNDA(q,a,p)
. | | | . | 12. i : ℤ
. | | | . | 13. 0≤i
. | | | . | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | 15. i = ||C||-1
. | | | . | 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | 17. ℕ1
. | | | . | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | | . |   by RWH
. | | | . |      (LemmaC
. | | | . |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | | . |       {length_append})
. | | | . |      0
. | | | . |      THEN
. | | | . |      Reduce 0
. | | | . |
. | | | . |\
. | | | . | 5. ||C||>0
. | | | . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . | 7. q : St
. | | | . | 8. a : Alph
. | | | . | 9. p : St
. | | | . | 10. NDA(C)⊢ q
. | | | . | 11. δNDA(q,a,p)
. | | | . | 12. i : ℤ
. | | | . | 13. 0≤i
. | | | . | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | 15. i = ||C||-1
```

```
. | | | . | 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | 17. ℕ1
. | | | . | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | | . |   by RWH
. | | | . |      (LemmaC
. | | | . |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | | . |       {length_append})
. | | | . |      0
. | | | . |      THEN
. | | | . |      Reduce 0
. | | | . |
. | | | . |\
. | | | . | 5. ||C||>0
. | | | . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . | 7. q : St
. | | | . | 8. a : Alph
. | | | . | 9. p : St
. | | | . | 10. NDA(C)⊢ q
. | | | . | 11. δNDA(q,a,p)
. | | | . | 12. i : ℤ
. | | | . | 13. 0≤i
. | | | . | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . | 15. i = ||C||-1
. | | | . | 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . | 17. ℕ1
. | | | . | ⊢  ||rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . | |            @ [<p,nil>])[i]))||≥1
. | | | . | | by RWH
. | | | . | |    (LemmaC
. | | | . | |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ
. | | | . | |     {length_reverse})
. | | | . | |    0
. | | | . | |
. | | | . | |\
. | | | . | | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | | . | |   by RWH
. | | | . | |      (LemmaC
. | | | . | |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | | . | |       {length_append})
. | | | . | |      0
. | | | . | |      THEN
. | | | . | |      Reduce 0
. | | | . | |
. | | | . | .\
. | | | . | . ⊢  ||2of((map(λc.<1of(c),a.2of(c)>;C)
. | | | . | . |        @ [<p,nil>])[i])||≥1
. | | | . | . | by RWH
. | | | . | . |    (LemmaC
. | | | . | . |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | | | . | . |     {select_append_front})
. | | | . | . |    0
. | | | . | . |
. | | | . | . ⊢  ||2of(map(λc.<1of(c),a.2of(c)>;C)[i])||≥1
. | | | . | .   by RWH
. | | | . | .      (LemmaC
```

```
. | | | . | .        Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | | . | .             map(f;as)[n] = f(as[n])
. | | | . | .        {map_select})
. | | | . | .       0
. | | | . | .       THEN
. | | | . | .       Reduce 0
. | | | . |
. | | | . .\
. | | | . . 5. ||C||>0
. | | | . . 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | | . . 7. q : St
. | | | . . 8. a : Alph
. | | | . . 9. p : St
. | | | . . 10. NDA(C)⊢ q
. | | | . . 11. δNDA(q,a,p)
. | | | . . 12. i : ℤ
. | | | . . 13. 0≤i
. | | | . . 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | | . . 15. i = ||C||-1
. | | | . . 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | | | . . 17. z : ℕ1
. | | | . . ⊢  1of([<p,nil>][z]) ∈ St  by Analyze 17
. | | | . . |
. | | | . . 17. z : ℤ
. | | | . . 18. 0 ≤ z < 1
. | | | . . ⊢
. | | | . . | by Inst
. | | | . . |    Thm* ∀a:T, as:T*, i:ℤ. i≤0 ⇒ (a.as)[i] = a
. | | | . . |    {select_cons_hd}
. | | | . . |    [St×Alph*;<p,nil>;nil;z]
. | | | . . |
. | | | . . 19. [<p,nil>][z] = <p,nil>
. | | | . . ⊢   by Analyze 19
. | | |
. | | .\
. | | . 15. ¬i = ||C||-1
. | | . ⊢
. | | . | by RWH
. | | . |    (LemmaC
. | | . |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | | . |     {select_append_front})
. | | . |    0

. | | . |
. | | . |\
. | | . | .....antecedent .....
. | | . | 5. ||C||>0
. | | . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | . | 7. q : St
. | | . | 8. a : Alph
. | | . | 9. p : St
. | | . | 10. NDA(C)⊢ q
. | | . | 11. δNDA(q,a,p)
. | | . | 12. i : ℤ
. | | . | 13. 0≤i
. | | . | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
```

```
. | | . | 15. ¬i = ||C||-1
. | | . | ⊢  ||rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | | . | |            @ [<p,nil>])[i]))||≥1
. | | . | | by RWH
. | | . | |    (LemmaC
. | | . | |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ
. | | . | |     {length_reverse})
. | | . | |    0
. | | . | |
. | | . | |\
. | | . | | ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | | . | |   by RWH
. | | . | |      (LemmaC
. | | . | |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | . | |       {length_append})
. | | . | |      0
. | | . | |      THEN
. | | . | |      Reduce 0
. | | . | |
. | | . | .\
. | | . | . ⊢  ||2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[i])||≥1
. | | . | . | by RWH
. | | . | . |    (LemmaC
. | | . | . |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | | . | . |     {select_append_front})
. | | . | . |    0
. | | . | . |
. | | . | . ⊢  ||2of(map(λc.<1of(c),a.2of(c)>;C)[i])||≥1
. | | . | . | by RWH
. | | . | . |    (LemmaC
. | | . | . |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | . | . |          map(f;as)[n] = f(as[n])
. | | . | . |     {map_select})
. | | . | . |    0
. | | . | . |    THEN
. | | . | . |    Reduce 0
. | | . | . |
. | | . | . ⊢  ||2of(C[i])||+1≥1  by Auto
. | | . |
. | | . |\
. | | . | 5. ||C||>0
. | | . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | . | 7. q : St
. | | . | 8. a : Alph
. | | . | 9. p : St
. | | . | 10. NDA(C)⊢ q
. | | . | 11. δNDA(q,a,p)
. | | . | 12. i : ℤ
. | | . | 13. 0≤i
. | | . | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | . | 15. ¬i = ||C||-1
. | | . | ⊢  i+1<||map(λc.<1of(c),a.2of(c)>;C)||  by Auto
. | | . |
. | | . .\
. | | . . ⊢  δNDA
```

```
. | | . . |  (1of(map(λc.<1of(c),a.2of(c)>;C)[i])
. | | . . |  ,hd(rev(2of(map(λc.<1of(c),a.2of(c)>;C)[i])))
. | | . . |  ,1of(map(λc.<1of(c),a.2of(c)>;C)[(i+1)]))
. | | . . | by RWH
. | | . . |    (LemmaC
. | | . . |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | . . |          map(f;as)[n] = f(as[n])
. | | . . |     {map_select})
. | | . . |    0
. | | . . |
. | | . . |\
. | | . . | .....antecedent .....
. | | . . | 5. ||C||>0
. | | . . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | . . | 7. q : St
. | | . . | 8. a : Alph
. | | . . | 9. p : St
. | | . . | 10. NDA(C)⊢ q
. | | . . | 11. δNDA(q,a,p)
. | | . . | 12. i : ℤ
. | | . . | 13. 0≤i
. | | . . | 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | . . | 15. ¬i = ||C||-1
. | | . . | ⊢  ||rev(2of(map(λc.<1of(c),a.2of(c)>;C)[i]))||≥1
. | | . . | | by RWH
. | | . . | |    (LemmaC
. | | . . | |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | | . . | |          map(f;as)[n] = f(as[n])
. | | . . | |     {map_select})
. | | . . | |    0
. | | . . | |
. | | . . | |\
. | | . . | | ⊢  i<||C||  by Auto
. | | . . | |
. | | . . | .\
. | | . . | . ⊢  ||rev(2of((λc.<1of(c),a.2of(c)>)(C[i])))||≥1
. | | . . | . | by Reduce 0
. | | . . | . |
. | | . . | . ⊢  ||rev(2of(C[i])) @ [a]||≥1
. | | . . | .   by RWH
. | | . . | .      (LemmaC
. | | . . | .       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. | | . . | .       {length_append})
. | | . . | .      0
. | | . . | .      THEN
. | | . . | .      Reduce 0
. | | . . |
. | | . . .\
. | | . . . ⊢  δNDA
. | | . . . |  (1of((λc.<1of(c),a.2of(c)>)(C[i]))
. | | . . . |  ,hd(rev(2of((λc.<1of(c),a.2of(c)>)(C[i]))))
. | | . . . |  ,1of((λc.<1of(c),a.2of(c)>)(C[(i+1)])))  by Reduce 0
. | | . . . |
. | | . . . ⊢  δNDA
. | | . . . |  (1of(C[i])
```

```
. | | . . . |  ,hd((rev(2of(C[i])) @ [a]))
. | | . . . |  ,1of(C[(i+1)]))  by Analyze 10
. | | . . . |
. | | . . . 10. I(NDA) = 1of(hd(C))
. | | . . . 11. (∀i:ℕ(||C||-1).
. | | . . . |     𝛅NDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . |     & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i])))))
. | | . . . |    & 1of(hd(rev(C))) = q
. | | . . . |    & 2of(hd(rev(C))) = nil
. | | . . . 12. 𝛅NDA(q,a,p)
. | | . . . 13. i : ℤ
. | | . . . 14. 0≤i
. | | . . . 15. i<||map(𝛌c.<1of(c),a.2of(c)>;C)||+1-1
. | | . . . 16. ¬i = ||C||-1
. | | . . . ⊢   by Analyze 11
. | | . . . |
. | | . . . 11. ∀i:ℕ(||C||-1).
. | | . . . |    𝛅NDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . |    & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | . . . 12. 1of(hd(rev(C))) = q & 2of(hd(rev(C))) = nil
. | | . . . 13. 𝛅NDA(q,a,p)
. | | . . . 14. i : ℤ
. | | . . . 15. 0≤i
. | | . . . 16. i<||map(𝛌c.<1of(c),a.2of(c)>;C)||+1-1
. | | . . . 17. ¬i = ||C||-1
. | | . . . ⊢   by Witness11 i
. | | . . . |
. | | . . . 11. 1of(hd(rev(C))) = q & 2of(hd(rev(C))) = nil
. | | . . . 12. 𝛅NDA(q,a,p)
. | | . . . 13. i : ℤ
. | | . . . 14. 0≤i
. | | . . . 15. i<||map(𝛌c.<1of(c),a.2of(c)>;C)||+1-1
. | | . . . 16. ¬i = ||C||-1
. | | . . . 17. 𝛅NDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . |    & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | . . . ⊢   by Analyze 17
. | | . . . |
. | | . . . 17. 𝛅NDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . 18. 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | . . . ⊢
. | | . . . | by RWH
. | | . . . |    (LemmaC
. | | . . . |     Thm* ∀l,m:T*. ||l||>0 ⇒ hd((l @ m)) = hd(l)
. | | . . . |     {hd_append_front})
. | | . . . |    0
. | | . . . |
. | | . . . |\
. | | . . . | 5. ||C||>0
. | | . . . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | | . . . | 7. q : St
. | | . . . | 8. a : Alph
. | | . . . | 9. p : St
. | | . . . | 10. I(NDA) = 1of(hd(C))
. | | . . . | 11. 1of(hd(rev(C))) = q
. | | . . . | 12. 2of(hd(rev(C))) = nil
```

```
. | | . . . | 13. δNDA(q,a,p)
. | | . . . | 14. i : ℤ
. | | . . . | 15. 0≤i
. | | . . . | 16. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | . . . | 17. ¬i = ||C||-1
. | | . . . | 18. δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . | 19. 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | . . . | ⊢  ||rev(2of(C[i]))||>0  by Witness6 i
. | | . . . | |
. | | . . . | |\
. | | . . . | | 6. q : St
. | | . . . | | 7. a : Alph
. | | . . . | | 8. p : St
. | | . . . | | 9. I(NDA) = 1of(hd(C))
. | | . . . | | 10. 1of(hd(rev(C))) = q
. | | . . . | | 11. 2of(hd(rev(C))) = nil
. | | . . . | | 12. δNDA(q,a,p)
. | | . . . | | 13. i : ℤ
. | | . . . | | 14. 0≤i
. | | . . . | | 15. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | . . . | | 16. ¬i = ||C||-1
. | | . . . | | 17. δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . | | 18. 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | . . . | | ⊢  i<||C||-1  by Auto
. | | . . . | |
. | | . . . | .\
. | | . . . | . 6. q : St
. | | . . . | . 7. a : Alph
. | | . . . | . 8. p : St
. | | . . . | . 9. I(NDA) = 1of(hd(C))
. | | . . . | . 10. 1of(hd(rev(C))) = q
. | | . . . | . 11. 2of(hd(rev(C))) = nil
. | | . . . | . 12. δNDA(q,a,p)
. | | . . . | . 13. i : ℤ
. | | . . . | . 14. 0≤i
. | | . . . | . 15. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | | . . . | . 16. ¬i = ||C||-1
. | | . . . | . 17. δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . | . 18. 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | | . . . | . 19. ||2of(C[i])||>0
. | | . . . | . ⊢
. | | . . . | . | by RWH
. | | . . . | . |    (LemmaC
. | | . . . | . |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ
. | | . . . | . |     {length_reverse})
. | | . . . | . |    0
. | | . . . | . |
. | | . . . | . ⊢  i<||C||
. | | . . . | .   by RWH
. | | . . . | .      (LemmaC
. | | . . . | .       Thm* ∀f:(A→B), as:A*.
. | | . . . | .             ||map(f;as)|| = ||as|| ∈ ℤ
. | | . . . | .       {map_length})
. | | . . . | .      15
. | | . . . |
```

```
. | | . . . . .\
. | | . . . . ⊢  δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | | . . . .   by Auto
. | |
. | .\
. | . 5. ||C||>0
. | . 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. | . 7. q : St
. | . 8. a : Alph
. | . 9. p : St
. | . 10. NDA(C)⊢ q
. | . 11. δNDA(q,a,p)
. | . 12. i : ℤ
. | . 13. 0≤i
. | . 14. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | . ⊢  2of((map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])[(i+1)])
. | . |  =
. | . |  rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | . |                 @ [<p,nil>])[i])))) by Decide (i<||C||-1)
. | . |
. | . |\
. | . | 15. i<||C||-1
. | . | ⊢
. | . | | by RWH
. | . | |    (LemmaC
. | . | |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | . | |     {select_append_front})
. | . | |    0
. | . | |
. | . | ⊢  2of(map(λc.<1of(c),a.2of(c)>;C)[(i+1)])
. | . | |  =
. | . | |  rev(tl(rev(2of(map(λc.<1of(c),a.2of(c)>;C)[i]))))
. | . | | by RWH
. | . | |    (LemmaC
. | . | |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | . | |          map(f;as)[n] = f(as[n])
. | . | |     {map_select})
. | . | |    0
. | . | |
. | . | ⊢  2of((λc.<1of(c),a.2of(c)>)(C[(i+1)]))
. | . | |  =
. | . | |  rev(tl(rev(2of((λc.<1of(c),a.2of(c)>)(C[i])))))
. | . | | by Reduce 0
. | . | |
. | . | ⊢  a.2of(C[(i+1)]) = rev(tl((rev(2of(C[i])) @ [a])))
. | . | | by RWH
. | . | |    (LemmaC
. | . | |     Thm* ∀l,m:T*. ||l||>0 ⇒ tl((l @ m)) = (tl(l) @ m)
. | . | |     {tl_append_front})
. | . | |    0
. | . | |
. | . | |\
. | . | | .....rewrite subgoal .....
. | . | | ⊢  ||rev(2of(C[i]))||>0  by Witness6 i
. | . | |
```

```
. | . | .\
. | . | . ⊢  a.2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))) @ [a])
. | . | . | by RWH
. | . | . |    (LemmaC
. | . | . |     Thm* ∀as,bs:T*. rev(as @ bs) = (rev(bs) @ rev(as))
. | . | . |     {reverse_append})
. | . | . |    0
. | . | . |
. | . | . ⊢  a.2of(C[(i+1)]) = (rev([a]) @ rev(tl(rev(2of(C[i])))))
. | . | . | by Reduce 0
. | . | . |
. | . | . ⊢  a.2of(C[(i+1)]) = a.rev(tl(rev(2of(C[i]))))  by Analyze
. | . | . |
. | . | . ⊢  2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))  by Analyze 10
. | . | . |
. | . | . 10. I(NDA) = 1of(hd(C))
. | . | . 11. (∀i:ℕ(||C||-1).
. | . | . |    δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | . | . |    & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i])))))
. | . | . |   & 1of(hd(rev(C))) = q
. | . | . |   & 2of(hd(rev(C))) = nil
. | . | . 12. δNDA(q,a,p)
. | . | . 13. i : ℤ
. | . | . 14. 0≤i
. | . | . 15. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | . | . 16. i<||C||-1
. | . | . ⊢   by Analyze 11
. | . | . |
. | . | . 11. ∀i:ℕ(||C||-1).
. | . | .     δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | . | .     & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | . | . 12. 1of(hd(rev(C))) = q & 2of(hd(rev(C))) = nil
. | . | . 13. δNDA(q,a,p)
. | . | . 14. i : ℤ
. | . | . 15. 0≤i
. | . | . 16. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | . | . 17. i<||C||-1
. | . | . ⊢   by Witness11 i
. | . |
. | . .\
. | . . 15. ¬i<||C||-1
. | . . ⊢
. | . . | by RW
. | . . |    (NthC 1
. | . . |     (LemmaC
. | . . |      Thm* ∀as,bs:T*, i:{||as||..(||as||+||bs||)⁻}.
. | . . |            (as @ bs)[i] = bs[(i-||as||)]
. | . . |      {select_append_back}))
. | . . |    0
. | . . |    THENA
. | . . |    (Auto THEN Reduce 0 THEN RemoveLabel)
. | . . |
. | . . ⊢  2of([<p,nil>][(i+1-||map(λc.<1of(c),a.2of(c)>;C)||)])
. | . . |  =
. | . . |  rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
```

```
. | . . |                 @ [<p,nil>])[i]))))
. | . . | by Assert (i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0)
. | . . |
. | . . 16. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | . . ⊢   by HypSubst 16 0
. | . . |
. | . . |\
. | . . | ⊢  2of([<p,nil>][0])
. | . . | |  =
. | . . | |  rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | . . | |                 @ [<p,nil>])[i]))))
. | . . | | by RW (NthC 1 (UnfoldTopC `select`)) 0
. | . . | |
. | . . | ⊢  2of(hd(nth_tl(0;[<p,nil>])))
. | . . | |  =
. | . . | |  rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | . . | |                 @ [<p,nil>])[i]))))
. | . . | | by RecCaseSplit `nth_tl`
. | . . | |
. | . . | |\
. | . . | | 17. 0≤0
. | . . | | ⊢  nil
. | . . | | |  =
. | . . | | |  rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | . . | | |                 @ [<p,nil>])[i]))))
. | . . | | | by RWH
. | . . | | |    (LemmaC
. | . . | | |     Thm* ∀as,bs:T*, i:ℕ||as||. (as @ bs)[i] = as[i]
. | . . | | |     {select_append_front})
. | . . | | |    0
. | . . | | |
. | . . | | ⊢  nil = rev(tl(rev(2of(map(λc.<1of(c),a.2of(c)>;C)[i]))))
. | . . | | | by RWH
. | . . | | |    (LemmaC
. | . . | | |     Thm* ∀f:(A→B), as:A*, n:ℕ||as||.
. | . . | | |          map(f;as)[n] = f(as[n])
. | . . | | |     {map_select})
. | . . | | |    0
. | . . | | |
. | . . | | ⊢  nil = rev(tl(rev(2of((λc.<1of(c),a.2of(c)>)(C[i])))))
. | . . | | | by Reduce 0
. | . . | | |
. | . . | | ⊢  nil = rev(tl((rev(2of(C[i])) @ [a])))
. | . . | | | by RWH
. | . . | | |    (LemmaC
. | . . | | |     Thm* ∀l,m:T*.
. | . . | | |          ||l|| = 0 ⇒ ||m||>0 ⇒ tl((l @ m)) = tl(m)
. | . . | | |     {tl_append_back})
. | . . | | |    0
. | . . | | |
. | . . | | |\
. | . . | | | .....rewrite subgoal .....
. | . . | | | ⊢  ||rev(2of(C[i]))|| = 0  by Analyze 10
. | . . | | | |
. | . . | | | 10. I(NDA) = 1of(hd(C))
```

```
. | . . | | | 11. (∀i:ℕ(||C||-1).
. | . . | | | |    δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | . . | | | |    & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i])))))
. | . . | | | |   & 1of(hd(rev(C))) = q
. | . . | | | |   & 2of(hd(rev(C))) = nil
. | . . | | | 12. δNDA(q,a,p)
. | . . | | | 13. i : ℤ
. | . . | | | 14. 0≤i
. | . . | | | 15. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | . . | | | 16. ¬i<||C||-1
. | . . | | | 17. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | . . | | | 18. 0≤0
. | . . | | | ⊢   by Analyze 11
. | . . | | | |
. | . . | | | 11. ∀i:ℕ(||C||-1).
. | . . | | | |   δNDA(1of(C[i]),hd(rev(2of(C[i]))),1of(C[(i+1)]))
. | . . | | | |   & 2of(C[(i+1)]) = rev(tl(rev(2of(C[i]))))
. | . . | | | 12. 1of(hd(rev(C))) = q & 2of(hd(rev(C))) = nil
. | . . | | | 13. δNDA(q,a,p)
. | . . | | | 14. i : ℤ
. | . . | | | 15. 0≤i
. | . . | | | 16. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | . . | | | 17. ¬i<||C||-1
. | . . | | | 18. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | . . | | | 19. 0≤0
. | . . | | | ⊢   by Analyze 12
. | . . | | | |
. | . . | | | 12. 1of(hd(rev(C))) = q
. | . . | | | 13. 2of(hd(rev(C))) = nil
. | . . | | | 14. δNDA(q,a,p)
. | . . | | | 15. i : ℤ
. | . . | | | 16. 0≤i
. | . . | | | 17. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | . . | | | 18. ¬i<||C||-1
. | . . | | | 19. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | . . | | | 20. 0≤0
. | . . | | | ⊢
. | . . | | | | by RWH
. | . . | | | |    (LemmaC
. | . . | | | |     Thm* ∀l:T*. ||l||>0 ⇒ hd(rev(l)) = l[(||l||-1)]
. | . . | | | |     {hd_reverse})
. | . . | | | |    13
. | . . | | | |
. | . . | | | 13. 2of(C[(||C||-1)]) = nil
. | . . | | | 14. δNDA(q,a,p)
. | . . | | | 15. i : ℤ
. | . . | | | 16. 0≤i
. | . . | | | 17. i<||map(λc.<1of(c),a.2of(c)>;C)||+1-1
. | . . | | | 18. ¬i<||C||-1
. | . . | | | 19. i+1-||map(λc.<1of(c),a.2of(c)>;C)|| = 0
. | . . | | | 20. 0≤0
. | . . | | | ⊢   by Assert (||C||-1 = i)
. | . . | | | |
. | . . | | | 21. ||C||-1 = i
. | . . | | | ⊢   by RevHypSubst 21 0
```

```
. | . . | | | |
. | . . | | | ⊢  ||rev(2of(C[(||C||-1)]))|| = 0  by HypSubst 13 0
. | . . | | | |
. | . . | | | ⊢  ||rev(nil)|| = 0  by Reduce 0
. | . . | | |
. | . . | | |\
. | . . | | | ⊢  ||nil||≥0  ...SupInf: lemma failure: non_neg_length
. | . . | | |   by Reduce 0 THEN RemoveLabel
. | . . | | |
. | . . | | .\
. | . . | | . ⊢  nil = rev(tl([a]))  by Reduce 0
. | . . | |
. | . . | .\
. | . . | . 17. 0<0
. | . . | . ⊢  2of("?")
. | . . | .    =
. | . . | .    rev(tl(rev(2of((map(λc.<1of(c),a.2of(c)>;C)
. | . . | .                   @ [<p,nil>])[i]))))  by Auto
. | . . |
. | . . |\
. | . . | 17. z : ℕ1
. | . . | ⊢  2of([<p,nil>][z]) ∈ Alph*  by Unfold ‘select‘ 0
. | . . | |
. | . . | ⊢  2of(hd(nth_tl(z;[<p,nil>]))) ∈ Alph*
. | . . |   by RecCaseSplit ‘nth_tl‘
. | . . |
. | . . .\
. | . . . 17. ℕ1
. | . . . ⊢  i<||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||
. | . . .   by Auto THEN Reduce 0 THEN RemoveLabel
. |
. .\
. . ⊢  1of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]))) = p
. . |  & 2of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]))) = nil
. . | by Analyze 0
. . |
. . |\
. . | ⊢  1of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]))) = p
. . | | by Analyze 4 THEN Analyze 4
. . | |
. . | 4. C : (St×Alph*)*
. . | 5. ||C||>0
. . | 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. . | 7. q : St
. . | 8. a : Alph
. . | 9. p : St
. . | 10. NDA(C)⊢ q
. . | 11. δNDA(q,a,p)
. . | ⊢
. . | | by RWH
. . | |    (LemmaC
. . | |     Thm* ∀as,bs:T*. rev(as @ bs) = (rev(bs) @ rev(as))
. . | |     {reverse_append})
. . | |    0
. . | |
```

```
. . | |\
. . | | .....antecedent .....
. . | | ⊢  ||rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])||≥1
. . | | | by RWH
. . | | |    (LemmaC
. . | | |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ    {length_reverse})
. . | | |    0
. . | | |
. . | | ⊢  ||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||≥1
. . | | | by RWH
. . | | |    (LemmaC
. . | | |     Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. . | | |     {length_append})
. . | | |    0
. . | | |
. . | | ⊢  ||map(λc.<1of(c),a.2of(c)>;C)||+||[<p,nil>]||≥1
. . | |   by Reduce 0
. . | |
. . | .\
. . | . ⊢  1of(hd((rev([<p,nil>])
. . | . |      @ rev(map(λc.<1of(c),a.2of(c)>;C)))))
. . | . |  =
. . | . |  p
. . | . | by RWH
. . | . |    (LemmaC
. . | . |     Thm* ∀l,m:T*. ||l||>0 ⇒ hd((l @ m)) = hd(l)
. . | . |     {hd_append_front})
. . | . |    0
. . | . |
. . | . |\
. . | . | .....rewrite subgoal .....
. . | . | ⊢  ||rev([<p,nil>])||>0  by Reduce 0
. . | . |
. . | . .\
. . | . . ⊢  1of(hd(rev([<p,nil>]))) = p
. . | . .   by RecUnfold `reverse` 0 THEN Reduce 0
. . |
. . .\
. . . ⊢  2of(hd(rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]))) = nil
. . . | by Analyze 4 THEN Analyze 4
. . . |
. . . 4. C : (St×Alph*)*
. . . 5. ||C||>0
. . . 6. ∀i:ℕ(||C||-1). ||2of(C[i])||>0
. . . 7. q : St
. . . 8. a : Alph
. . . 9. p : St
. . . 10. NDA(C)⊢ q
. . . 11. δNDA(q,a,p)
. . . ⊢
. . . | by RWH
. . . |    (LemmaC
. . . |     Thm* ∀as,bs:T*. rev(as @ bs) = (rev(bs) @ rev(as))
. . . |     {reverse_append})
. . . |    0
```

```
. . . |
. . . |\
. . . | .....antecedent .....
. . . | ⊢  ||rev(map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>])||≥1
. . . | | by RWH
. . . | |    (LemmaC
. . . | |     Thm* ∀l:T*. ||rev(l)|| = ||l|| ∈ ℤ    {length_reverse})
. . . | |    0
. . . | |
. . . | ⊢  ||map(λc.<1of(c),a.2of(c)>;C) @ [<p,nil>]||≥1
. . . |   by RWH
. . . |      (LemmaC
. . . |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
. . . |       {length_append})
. . . |      0
. . . |      THEN
. . . |      Reduce 0
. . . |
. . . .\
. . . . ⊢  2of(hd((rev([<p,nil>])
. . . . |      @ rev(map(λc.<1of(c),a.2of(c)>;C)))))
. . . . |  =
. . . . |  nil
. . . . | by RWH
. . . . |    (LemmaC
. . . . |     Thm* ∀l,m:T*. ||l||>0 ⇒ hd((l @ m)) = hd(l)
. . . . |     {hd_append_front})
. . . . |    0
. . . . |
. . . . |\
. . . . | .....rewrite subgoal .....
. . . . | ⊢  ||rev([<p,nil>])||>0
. . . . |   by RecUnfold ‘reverse‘ 0 THEN Reduce 0
. . . . |
. . . . .\
. . . . . ⊢  2of(hd(rev([<p,nil>]))) = nil
. . . . .   by RecUnfold ‘reverse‘ 0 THEN Reduce 0

END
```