```
atomic_char  THM

⊢  ∀a:ℤ. atomic(a) ⟺ ¬(a ∼ 1) & (∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a))
| by Unfolds [`guard`;`atomic`] 0 THEN GenExRepD
|
|\
| 1. a : ℤ
| 2. ¬a = 0
| 3. ¬(a ∼ 1)
| 4. ¬reducible(a)
| 5. b : ℤ
| 6. b | a
| ⊢  (b ∼ 1) ∨ (b ∼ a)
| | by (NegateConcl2
| |     THENA
| |     BackThru Thm* Dec(P) ⇒ Stable{P}    {stable__from_decidable})
| |    THEN
| |    (RWH (LemmaC Thm* ¬(A ∨ B) ⟺ ¬A & ¬B    {not_over_or}) -1)
| |    THEN
| |    (Analyze -1)
| |
| 7. ¬(b ∼ 1)
| 8. ¬(b ∼ a)
| ⊢  False
| | by Witness6 c THEN (Analyze 4) THEN (Unfold `reducible` 0)
| |
| 4. b : ℤ
| 5. c : ℤ
| 6. a = b•c
| 7. ¬(b ∼ 1)
| 8. ¬(b ∼ a)
| ⊢  ∃b,c:ℤ^-0. ¬(b ∼ 1) & ¬(c ∼ 1) & a = b•c
| | by (CopyToEnd 2) THEN (HypSubst 6 -1)
| |    THEN
| |    (Simple
| |     (FwdThru
| |      Thm* ∀a,b:ℤ. ¬a•b = 0 ⇒ ¬a = 0 & ¬b = 0    {zero_ann_b}
| |      [-1]))
| |    THEN
| |    (Analyze -1)
| |
| 9. ¬b•c = 0
| 10. ¬b = 0
| 11. ¬c = 0
| ⊢  ∃b,c:ℤ^-0. ¬(b ∼ 1) & ¬(c ∼ 1) & a = b•c  by InstConcl [b;c]
| |
| ⊢  ¬(c ∼ 1)  by (HypSubst 6 8) THEN (Analyze 0)
| |
| 8. ¬(b ∼ (b•c))
| 9. ¬b•c = 0
| 10. ¬b = 0
| 11. ¬c = 0
| 12. c ∼ 1
| ⊢  False  by (RWH (HypC 12) 8) THEN (Analyze 8) THEN (ArithSimp 0)
| |
```

```
| 8. ¬b•c = 0
| 9. ¬b = 0
| 10. ¬c = 0
| 11. c ∼ 1
| ⊢  b ∼ b  by RelRST
|
|\
| 1. a : ℤ
| 2. ¬(a ∼ 1)
| 3. ∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a)
| ⊢  ¬a = 0  by Witness3 2 THEN (Analyze 0)
| |
| 3. 2 | a ⇒ (2 ∼ 1) ∨ (2 ∼ a)
| 4. a = 0
| ⊢  False  by HypSubst 4 3 THEN Analyze 3
| |
| |\
| | .....antecedent .....
| | 3. a = 0
| | ⊢  2 | 0  by BackThru Thm* ∀b:ℤ. b | 0    {any_divs_zero}
| |
| .\
| . 3. a = 0
| . 4. (2 ∼ 1) ∨ (2 ∼ 0)
| . ⊢  False
| .   by (RWH
| .       (LemmaC
| .        Thm* ∀a,b:ℤ. (a ∼ b) ⟺ a = b ∨ a = -b    {assoced_elim})
| .       4)
| .      THEN
| .      GenExRepD
|
.\
. 1. a : ℤ
. 2. ¬(a ∼ 1)
. 3. ∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a)
. ⊢  ¬reducible(a)
. | by (Analyze 0) THEN (Unfold ‘reducible‘ -1) THEN ExRepD
. |
. 4. b : ℤ⁻⁰
. 5. c : ℤ⁻⁰
. 6. ¬(b ∼ 1)
. 7. ¬(c ∼ 1)
. 8. a = b•c
. ⊢  False  by (InstHyp [b] 3) THEN (InstHyp [c] 3) THEN GenExRepD
. |
. |\
. | .....antecedent .....
. | ⊢  b | a  by Witness c
. |
. |\
. | .....antecedent .....
. | 9. (b ∼ 1) ∨ (b ∼ a)
. | ⊢  c | a  by Witness b
. |
```

```
. .\
. . 9. b ∼ a
. . 10. c ∼ a
. . ⊢  False  by Assert ((a•1) ∼ (b•c)) THENA StrengthenRel
. . |
. . 11. (a•1) ∼ (b•c)
. . ⊢  False  by RWH (HypC 9) 11
. . |
. . 11. (a•1) ∼ (a•c)
. . ⊢  False
. . | by FwdThru
. . |    Thm* ∀a,b:ℤ, n:ℤ⁻⁰. ((n•a) ∼ (n•b)) ⇒ (a ∼ b)
. . |    {mul_cancel_in_assoced}
. . |    [11]
. . |
. . |\
. . | .....set predicate .....
. . | ⊢  a ≠ 0  by (Analyze 0) THEN (HypSubst 12 8)
. . | |
. . | 8. 0 = b•c
. . | 9. b ∼ a
. . | 10. c ∼ a
. . | 11. (a•1) ∼ (a•c)
. . | 12. a = 0
. . | ⊢  False
. . |   by (InvertRel 8)
. . |      THEN
. . |      (FwdThru
. . |       Thm* ∀a,b:ℤ. a•b = 0 ⇒ a = 0 ∨ b = 0    {int_entire}
. . |       [8])
. . |      THEN
. . |      (Analyze -1)
. . |
. . .\
. . . 12. 1 ∼ c
. . . ⊢  False  by InvertRel 12
END
```