```
atomic_imp_prime  THM

⊢  ∀a:ℤ. atomic(a) ⇒ prime(a)
| by UnivCD
|    THEN
|    (FwdThru
|     Thm* ∀a:ℤ.
|          atomic(a) ⟺ ¬(a ∼ 1) & (∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a))
|     {atomic_char}
|     [-1])
|
1. a : ℤ
2. atomic(a)
3. ¬(a ∼ 1) & (∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a))
⊢  prime(a)
| by Unfold ‘prime‘ 0 THEN Unfold ‘atomic‘ 2 THEN GenUnivCD
|    THEN
|    (Complete Auto ORELSE Thin 2)
|
2. ¬(a ∼ 1) & (∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a))
3. b : ℤ
4. c : ℤ
5. a | b·c
⊢  a | b ∨ a | c  by Analyze 2
|
2. ¬(a ∼ 1)
3. ∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a)
4. b : ℤ
5. c : ℤ
6. a | b·c
⊢  a | b ∨ a | c
| by ((InstHyp [gcd(a;b)] 3) THEN (InstHyp [gcd(a;c)] 3))
|    THENA
|    Try (BackThru Thm* ∀a,b:ℤ. gcd(a;b) | a    {gcd_is_divisor_1})
|
7. (gcd(a;b) ∼ 1) ∨ (gcd(a;b) ∼ a)
8. (gcd(a;c) ∼ 1) ∨ (gcd(a;c) ∼ a)
⊢  a | b ∨ a | c  by Analyze 8
|
|\
| 8. gcd(a;c) ∼ 1
| ⊢  a | b ∨ a | c  by Analyze 7
| |
| |\
| | 7. gcd(a;b) ∼ 1
| | 8. gcd(a;c) ∼ 1
| | ⊢  a | b ∨ a | c
| | | by (FwdThru
| | |     Thm* ∀a,b:ℤ. CoPrime(a,b) ⟺ (gcd(a;b) ∼ 1)
| | |     {coprime_elim_a}
| | |     [7])
| | |    THEN
| | |    (FwdThru
| | |     Thm* ∀a,b:ℤ. CoPrime(a,b) ⟺ (gcd(a;b) ∼ 1)
| | |     {coprime_elim_a}
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| | |     [8])
| | |    THEN
| | |    (FwdThru
| | |     Thm* ∀a,b1,b2:ℤ.
| | |          CoPrime(a,b1) ⇒ CoPrime(a,b2) ⇒ CoPrime(a,b1·b2)
| | |     {coprime_prod}
| | |     [9;10])
| | |
| | 9. CoPrime(a,b)
| | 10. CoPrime(a,c)
| | 11. CoPrime(a,b·c)
| | ⊢  a | b ∨ a | c  by RepUnfolds [‘coprime‘;‘gcd_p‘] 11 THEN RepD
| | |
| | 11. 1 | a
| | 12. 1 | b·c
| | 13. ∀z:ℤ. z | a & z | b·c ⇒ z | 1
| | ⊢  a | b ∨ a | c  by InstHyp [a] 13
| | |
| | |\
| | | ⊢  a | a  by RelRST
| | |
| | .\
| | . 14. a | 1
| | . ⊢  a | b ∨ a | c  by (Analyze 2) THEN (Analyze 0)
| |
| .\
| . 7. gcd(a;b) ∼ a
| . 8. gcd(a;c) ∼ 1
| . ⊢  a | b ∨ a | c
| .   by (Sel 1 (Analyze 0)) THEN (RWH (RevHypC 7) 0)
| .      THEN
| .      (BackThru Thm* ∀a,b:ℤ. gcd(a;b) | b    {gcd_is_divisor_2})
|
.\
. 8. gcd(a;c) ∼ a
. ⊢  a | b ∨ a | c
.   by (Sel 2 (Analyze 0)) THEN (RWH (RevHypC 8) 0)
.      THEN
.      (BackThru Thm* ∀a,b:ℤ. gcd(a;b) | b    {gcd_is_divisor_2})
END
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