```
bezout_ident_n  THM

⊢  ∀b:ℕ, a:ℤ. ∃u,v:ℤ. GCD(a;b;u·a+v·b)
| by (Analyze 0) THEN (OnVar ‘b’ CompNatInd) THEN (Analyze 0)
|
1. b : ℕ
2. ∀b1:ℕ. b1<b ⇒ (∀a:ℤ. ∃u,v:ℤ. GCD(a;b1;u·a+v·b1))
3. a : ℤ
⊢  ∃u,v:ℤ. GCD(a;b;u·a+v·b)  by Decide (b = 0)
|
|\
| 4. b = 0
| ⊢  ∃u,v:ℤ. GCD(a;b;u·a+v·b)
| | by (InstConcl [1;0]) THEN (HypSubst 4 0) THEN (ArithSimp 0)
| |
| ⊢  GCD(a;0;a)  by BackThru Thm* ∀a:ℤ. GCD(a;0;a)    {gcd_p_zero}
|
.\
. 4. ¬b = 0
. ⊢  ∃u,v:ℤ. GCD(a;b;u·a+v·b)
. | by Inst
. |    Thm* ∀a:ℤ, b:ℕ⁺. ∃q:ℤ, r:ℕb. a = q·b+r    {quot_rem_exists}
. |    [a;b]
. |    THEN
. |    ExistHD -1
. |
. 5. q : ℤ
. 6. r : ℕb
. 7. a = q·b+r
. ⊢  ∃u,v:ℤ. GCD(a;b;u·a+v·b)  by InstHyp [r;b] 2 THEN ExistHD -1
. |
. 8. u : ℤ
. 9. v : ℤ
. 10. GCD(b;r;u·b+v·r)
. ⊢  ∃u,v:ℤ. GCD(a;b;u·a+v·b)
. | by (InstConcl [v;u-q·v]) THEN (HypSubst 7 0)
. |    THEN
. |    (OnMCls [0;10] ArithSimp)
. |
. 10. GCD(b;r;b·u+r·v)
. ⊢  GCD(r+b·q;b;b·u+r·v)
. | by BackThru
. |    Thm* ∀a,b,y:ℤ. GCD(a;b;y) ⇒ GCD(b;a;y)    {gcd_p_sym}
. |
. ⊢  GCD(b;r+b·q;b·u+r·v)
.   by (RWN 1 (LemmaC Thm* ∀a,b:ℤ. a·b = b·a    {mul_com}) 0)
.      THEN
.      (BackThru
.       Thm* ∀a,b,y,k:ℤ. GCD(a;b;y) ⇒ GCD(a;b+k·a;y)
.       {gcd_p_shift})
END
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