```
coprime_prod  THM

⊢  ∀a,b1,b2:ℤ. CoPrime(a,b1) ⇒ CoPrime(a,b2) ⇒ CoPrime(a,b1·b2)
| by UnivCD
|
1. a : ℤ
2. b1 : ℤ
3. b2 : ℤ
4. CoPrime(a,b1)
5. CoPrime(a,b2)
⊢  CoPrime(a,b1·b2)
| by (FwdThru
|      Thm* ∀a,b:ℤ. CoPrime(a,b) ⟺ (∃x,y:ℤ. a·x+b·y = 1)
|      {coprime_bezout_id}
|      [4])
|     THEN
|     (FwdThru
|      Thm* ∀a,b:ℤ. CoPrime(a,b) ⟺ (∃x,y:ℤ. a·x+b·y = 1)
|      {coprime_bezout_id}
|      [5])
|     THEN
|     (OnHyps [5;4] Thin)
|     THEN
|     (BackThru
|      Thm* ∀a,b:ℤ. CoPrime(a,b) ⟺ (∃x,y:ℤ. a·x+b·y = 1)
|      {coprime_bezout_id})
|
4. ∃x,y:ℤ. a·x+b1·y = 1
5. ∃x,y:ℤ. a·x+b2·y = 1
⊢  ∃x,y:ℤ. a·x+b1·b2·y = 1
| by OnHyps [5;6;4;5] Analyze THEN RenameVar ‘x2’ 7
|     THEN
|     RenameVar ‘y2’ 8
|
4. x1 : ℤ
5. y1 : ℤ
6. a·x1+b1·y1 = 1
7. x2 : ℤ
8. y2 : ℤ
9. a·x2+b2·y2 = 1
⊢  ∃x,y:ℤ. a·x+b1·b2·y = 1
| by (Using [‘n’,(-a·x1)]
|      (FwdThru Thm* ∀a,b,n:ℤ. a = b ⟺ a+n = b+n    {add_mono_wrt_eq}
|       [6]))
|     THEN
|     (Using [‘n’,(-a·x2)]
|      (FwdThru Thm* ∀a,b,n:ℤ. a = b ⟺ a+n = b+n    {add_mono_wrt_eq}
|       [9]))
|     THEN
|     (OnHyps [9;6] Thin)
|     THEN
|     (ArithSimp 8)
|     THEN
|     (ArithSimp 9)
|
```

```
6. x2 : ℤ
7. y2 : ℤ
8. b1·y1 = 1+-a·x1
9. b2·y2 = 1+-a·x2
⊢  ∃x,y:ℤ. a·x+b1·b2·y = 1  by InstConcl [x1+x2-a·x1·x2;y1·y2]
|
⊢  a·(x1+x2-a·x1·x2)+b1·b2·y1·y2 = 1
| by (Using [‘n’,((1-a·x1)·(1-a·x2)-1)]
|     (BackThru
|      Thm* ∀a,b,n:ℤ. a = b ⟺ a+n = b+n    {add_mono_wrt_eq}))
|    THEN
|    (ArithSimp 0)
|
⊢  b1·b2·y1·y2 = 1+a·a·x1·x2+-a·x1+-a·x2
  by FwdThru
     Thm* ∀i1,i2,j1,j2:ℤ. i1 = j1 ⇒ i2 = j2 ⇒ i1·i2 = j1·j2
     {mul_functionality_wrt_eq}
     [8;9]
END
```