```
eqmod_functionality_wrt_eqmod  THM

⊢  ∀m,m',a,a',b,b':ℤ.
|  m = m'
|  ⇒
|  (a = a' mod m)
|  ⇒
|  (b = b' mod m) ⇒ ((a = b mod m) ⟺ (a' = b' mod m'))
| by GenUnivCD
|
|\
| 1. m : ℤ
| 2. m' : ℤ
| 3. a : ℤ
| 4. a' : ℤ
| 5. b : ℤ
| 6. b' : ℤ
| 7. m = m'
| 8. a = a' mod m
| 9. b = b' mod m
| 10. a = b mod m
| ⊢  a' = b' mod m'  by (RevHypSubst 7 0) THEN (OnHyps [7;2] Thin)
| |
| 2. a : ℤ
| 3. a' : ℤ
| 4. b : ℤ
| 5. b' : ℤ
| 6. a = a' mod m
| 7. b = b' mod m
| 8. a = b mod m
| ⊢  a' = b' mod m
| | by (FwdThru
| |     Thm* ∀m,a,b:ℤ. (a = b mod m) ⇒ (b = a mod m)
| |     {eqmod_inversion}
| |     [6])
| |    THEN
| |    (Thin 6)
| |
| 6. b = b' mod m
| 7. a = b mod m
| 8. a' = a mod m
| ⊢  a' = b' mod m
|   by (FwdThru
|       Thm* ∀m,a,b,c:ℤ.
|            (a = b mod m) ⇒ (b = c mod m) ⇒ (a = c mod m)
|       {eqmod_transitivity}
|       [8;7])
|      THEN
|      (FwdThru
|       Thm* ∀m,a,b,c:ℤ.
|            (a = b mod m) ⇒ (b = c mod m) ⇒ (a = c mod m)
|       {eqmod_transitivity}
|       [6;9])
|
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. 1. m : ℤ
. 2. m' : ℤ
. 3. a : ℤ
. 4. a' : ℤ
. 5. b : ℤ
. 6. b' : ℤ
. 7. m = m'
. 8. a = a' mod m
. 9. b = b' mod m
. 10. a' = b' mod m'
. ⊢  a = b mod m  by (RevHypSubst 7 10) THEN (OnHyps [7;2] Thin)
. |
. 2. a : ℤ
. 3. a' : ℤ
. 4. b : ℤ
. 5. b' : ℤ
. 6. a = a' mod m
. 7. b = b' mod m
. 8. a' = b' mod m
. ⊢  a = b mod m
. | by (FwdThru
. |     Thm* ∀m,a,b:ℤ. (a = b mod m) ⇒ (b = a mod m)
. |     {eqmod_inversion}
. |     [7])
. |    THEN
. |    (Thin 7)
. |
. 7. a' = b' mod m
. 8. b' = b mod m
. ⊢  a = b mod m
.   by (FwdThru
.       Thm* ∀m,a,b,c:ℤ.
.            (a = b mod m) ⇒ (b = c mod m) ⇒ (a = c mod m)
.       {eqmod_transitivity}
.       [6;7])
.      THEN
.      (FwdThru
.       Thm* ∀m,a,b,c:ℤ.
.            (a = b mod m) ⇒ (b = c mod m) ⇒ (a = c mod m)
.       {eqmod_transitivity}
.       [9;8])
END
```