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gcd_ex_n  THM

⊢  ∀b:ℕ, a:ℤ. ↓∃y:ℤ. GCD(a;b;y)
| by Analyze 0 THEN ((OnVar ‘b’ CompNatInd) THEN (Analyze 0))
|
1. b : ℕ
2. ∀b1:ℕ. b1<b ⇒ (∀a:ℤ. ↓∃y:ℤ. GCD(a;b1;y))
3. a : ℤ
⊢  ↓∃y:ℤ. GCD(a;b;y)  by Decide (b = 0)
|
|\
| 4. b = 0
| ⊢  ↓∃y:ℤ. GCD(a;b;y)  by Witness a THEN (RWH (HypC 4) 0)
| |
| ⊢  GCD(a;0;a)  by BackThru Thm* ∀a:ℤ. GCD(a;0;a)    {gcd_p_zero}
|
.\
. 4. ¬b = 0
. ⊢  ↓∃y:ℤ. GCD(a;b;y)
. | by (Inst
. |     Thm* ∀a:ℤ, b:ℕ+. ∃q:ℤ, r:ℕb. a = q·b+r    {quot_rem_exists}
. |     [a;b])
. |    THEN
. |    (ExistHD -1)
. |
. 5. q : ℤ
. 6. r : ℕb
. 7. a = q·b+r
. ⊢  ↓∃y:ℤ. GCD(a;b;y)  by InstHyp [r;b] 2
. |
. 8. ↓∃y:ℤ. GCD(b;r;y)
. ⊢  ↓∃y:ℤ. GCD(a;b;y)  by (Analyze -1) THEN Witness y
. |
. 8. y : ℤ
. 9. GCD(b;r;y)
. ⊢  GCD(a;b;y)
. | by (BackThru
. |     Thm* ∀a,b,y:ℤ. GCD(a;b;y) ⇒ GCD(b;a;y)    {gcd_p_sym})
. |    THEN
. |    (RWH (HypC 7) 0)
. |
. ⊢  GCD(b;q·b+r;y)
. | by RWH
. |    (LemmaC Thm* ∀a,b:ℤ. a+b = b+a    {add_com}
. |     ORELSEC
. |     LemmaC Thm* ∀a,b:ℤ. a·b = b·a    {mul_com})
. |    0
. |
. ⊢  GCD(b;r+q·b;y)
.   by BackThru
.      Thm* ∀a,b,y,k:ℤ. GCD(a;b;y) ⇒ GCD(a;b+k·a;y)    {gcd_p_shift}
END
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