```
gcd_p_mul  THM

⊢  ∀a,b,y,n:ℤ. GCD(a;b;y) ⇒ GCD(n•a;n•b;n•y)
| by RWN 2 (UnfoldTopC ‘gcd_p‘) 0 THEN GenUnivCD
|
|\
| 1. a : ℤ
| 2. b : ℤ
| 3. y : ℤ
| 4. n : ℤ
| 5. GCD(a;b;y)
| ⊢  n•y | n•a
|   by UnfoldTopAb -1
|      THEN
|      Backchain
|      [Thm* ∀a,b:ℤ. a | b ⇒ (∀n:ℤ. n•a | n•b)    {divides_mul}]
|
|\
| 1. a : ℤ
| 2. b : ℤ
| 3. y : ℤ
| 4. n : ℤ
| 5. GCD(a;b;y)
| ⊢  n•y | n•b
|   by UnfoldTopAb -1
|      THEN
|      Backchain
|      [Thm* ∀a,b:ℤ. a | b ⇒ (∀n:ℤ. n•a | n•b)    {divides_mul}]
|
.\
. 1. a : ℤ
. 2. b : ℤ
. 3. y : ℤ
. 4. n : ℤ
. 5. GCD(a;b;y)
. 6. z : ℤ
. 7. z | n•a & z | n•b
. ⊢  z | n•y
. | by (Inst Thm* ∀a,b:ℤ. ∃u,v:ℤ. GCD(a;b;u•a+v•b)    {bezout_ident}
. |     [a;b])
. |    THEN
. |    ExRepD
. |    THEN
. |    (FwdThru
. |     Thm* ∀a,b,y1,y2:ℤ. GCD(a;b;y1) ⇒ GCD(a;b;y2) ⇒ (y1 ∼ y2)
. |     {gcd_unique}
. |     [5;-1])
. |
. 7. z | n•a
. 8. z | n•b
. 9. u : ℤ
. 10. v : ℤ
. 11. GCD(a;b;u•a+v•b)
. 12. y ∼ (u•a+v•b)
. ⊢  z | n•y  by RWH (HypC 12) 0
```

```
. |
. ⊢  z | n·(u·a+v·b)
. | by (Using [`c`,u]
. |     (FwdThru Thm* ∀a,b,c:ℤ. a | b ⇒ a | b·c    {divisor_of_mul}
. |      [7]))
. |    THEN
. |    (Using [`c`,v]
. |     (FwdThru Thm* ∀a,b,c:ℤ. a | b ⇒ a | b·c    {divisor_of_mul}
. |      [8]))
. |
. 13. z | n·a·u
. 14. z | n·b·v
. ⊢  z | n·(u·a+v·b)
.   by (FwdThru
.       Thm* ∀a,b1,b2:ℤ. a | b1 ⇒ a | b2 ⇒ a | b1+b2
.       {divisor_of_sum}
.       [13;14])
.      THEN
.      (OnMCls [-1;0] ArithSimp)
END
```