```
gcd_p_shift  THM

⊢  ∀a,b,y,k:ℤ. GCD(a;b;y) ⇒ GCD(a;b+k•a;y)  by Unfold ‘gcd_p‘ 0
|
|\
| 1. a : ℤ
| 2. b : ℤ
| 3. y : ℤ
| 4. k : ℤ
| 5. y | a
| 6. y | b
| 7. ∀z:ℤ. z | a & z | b ⇒ z | y
| ⊢  y | b+k•a
| | by BackThru
| |    Thm* ∀a,b1,b2:ℤ. a | b1 ⇒ a | b2 ⇒ a | b1+b2
| |    {divisor_of_sum}
| |
| |\
| | ⊢  y | b  by Trivial
| |
| .\
| . ⊢  y | k•a
| . | by Rewrite
| . |    (HigherC (LemmaC Thm* ∀a,b:ℤ. a•b = b•a    {mul_com}))
| . |    0
| . |
| . ⊢  y | a•k
| .   by BackThru Thm* ∀a,b,c:ℤ. a | b ⇒ a | b•c    {divisor_of_mul}
|
.\
. 1. a : ℤ
. 2. b : ℤ
. 3. y : ℤ
. 4. k : ℤ
. 5. y | a
. 6. y | b
. 7. ∀z:ℤ. z | a & z | b ⇒ z | y
. 8. z : ℤ
. 9. z | a
. 10. z | b+k•a
. ⊢  z | y  by InstHyp [z] 7
. |
. |\
. | ⊢  z | b  by Assert (z | b+k•a+a•(-k))
. | |
. | |\
. | | .....assertion .....
. | | ⊢  z | b+k•a+a•(-k)
. | | | by BackThru
. | | |    Thm* ∀a,b1,b2:ℤ. a | b1 ⇒ a | b2 ⇒ a | b1+b2
. | | |    {divisor_of_sum}
. | | |
. | | |\
. | | | ⊢  z | b+k•a  by Hypothesis
. | | |
```

```
. | | .\
. | | . ⊢  z | a•(-k)
. | | .   by BackThru
. | | .      Thm* ∀a,b,c:ℤ. a | b ⇒ a | b•c    {divisor_of_mul}
. | |
. | .\
. | . 11. z | b+k•a+a•(-k)
. | . ⊢  z | b  by ArithSimp 11
. |
. .\
. . 11. z | y
. . ⊢  z | y  by Trivial
END
```