```
gcd_sat_pred  THM

⊢  ∀a,b:ℤ. GCD(a;b;gcd(a;b))
| by SeqOnM
|    [UnivCD
|    ;OnVar ‘a’ MoveToConcl
|    ;OnVar ‘b’ (RankInd (λi.|i|) ℕ CompNatInd)
|    ;Analyze 0]
|
1. b : ℤ
2. ∀b1:ℤ. |b1|<|b| ⇒ (∀a:ℤ. GCD(a;b1;gcd(a;b1)))
3. a : ℤ
⊢  GCD(a;b;gcd(a;b))  by RecCaseSplit ‘gcd‘
|
|\
| .....truecase .....
| 4. b = 0
| ⊢  GCD(a;b;a)
|   by (HypSubst 4 0)
|      THEN
|      (BackThru Thm* ∀a:ℤ. GCD(a;0;a)    {gcd_p_zero})
|
.\
. .....falsecase .....
. 4. ¬b = 0
. ⊢  GCD(a;b;gcd(b;a rem b))  by InstHyp [a rem b;b] 2
. |
. |\
. | .....antecedent .....
. | ⊢  |a rem b|<|b|
. |   by BackThru Thm* ∀a:ℤ, b:ℤ^-0. |a rem b|<|b|    {rem_bounds_z}
. |
. .\
. . 5. GCD(b;a rem b;gcd(b;a rem b))
. . ⊢  GCD(a;b;gcd(b;a rem b))
. . | by SeqOnM
. . |    [RWN 1
. . |     (LemmaC
. . |      Thm* ∀a:ℤ, n:ℤ^-0. (a rem n) = a-(a ÷ n)·n    {rem_to_div})
. . |     5
. . |    ;Using [‘k’,(a ÷ b)]
. . |     (FwdThru
. . |      Thm* ∀a,b,y,k:ℤ. GCD(a;b;y) ⇒ GCD(a;b+k·a;y)
. . |      {gcd_p_shift}
. . |      [5])
. . |    ;ArithSimp 6]
. . |
. . 5. GCD(b;a-(a ÷ b)·b;gcd(b;a rem b))
. . 6. GCD(b;a;gcd(b;a rem b))
. . ⊢  GCD(a;b;gcd(b;a rem b))
. .   by BackThru
. .      Thm* ∀a,b,y:ℤ. GCD(a;b;y) ⇒ GCD(b;a;y)    {gcd_p_sym}
END
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