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multiply_functionality_wrt_assoced  THM

⊢  ∀a,a',b,b':ℤ. (a ∼ a') ⇒ (b ∼ b') ⇒ ((a•b) ∼ (a'•b'))
| by Unfold ‘assoced‘ 0 THEN GenRepD
|
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| 1. a : ℤ
| 2. a' : ℤ
| 3. b : ℤ
| 4. b' : ℤ
| 5. a | a'
| 6. a' | a
| 7. b | b'
| 8. b' | b
| ⊢  a•b | a'•b'  by Analyze 7 THEN Analyze 5
| |
| 5. c1 : ℤ
| 6. a' = a•c1
| 7. a' | a
| 8. c : ℤ
| 9. b' = b•c
| 10. b' | b
| ⊢  a•b | a'•b'
|   by Witness c•c1 THEN (HypSubst 6 0) THEN (HypSubst 9 0)
|
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. 1. a : ℤ
. 2. a' : ℤ
. 3. b : ℤ
. 4. b' : ℤ
. 5. a | a'
. 6. a' | a
. 7. b | b'
. 8. b' | b
. ⊢  a'•b' | a•b  by Analyze 8 THEN Analyze 6
. |
. 6. c1 : ℤ
. 7. a = a'•c1
. 8. b | b'
. 9. c : ℤ
. 10. b = b'•c
. ⊢  a'•b' | a•b
.   by Witness c•c1 THEN (HypSubst 7 0) THEN (HypSubst 10 0)
END
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