```
quot_rem_exists  THM

⊢  ∀a:ℤ, b:ℕ+. ∃q:ℤ, r:ℕb. a = q·b+r  by UnivCD
|
1. a : ℤ
2. b : ℕ+
⊢  ∃q:ℤ, r:ℕb. a = q·b+r  by Decide (0≤a)
|
|\
| 3. 0≤a
| ⊢  ∃q:ℤ, r:ℕb. a = q·b+r
| | by Inst
| |    Thm* ∀a:ℕ, b:ℕ+. ∃q:ℕ, r:ℕb. a = q·b+r    {quot_rem_exists_n}
| |    [a;b]
| |
| 4. ∃q:ℕ, r:ℕb. a = q·b+r
| ⊢  ∃q:ℤ, r:ℕb. a = q·b+r  by Analyze 4 THEN Witness q
|
.\
. 3. ¬0≤a
. ⊢  ∃q:ℤ, r:ℕb. a = q·b+r
. | by (Inst
. |     Thm* ∀a:ℕ, b:ℕ+. ∃q:ℕ, r:ℕb. a = q·b+r    {quot_rem_exists_n}
. |     [-a;b])
. |    THEN
. |    (ExistHD -1)
. |
. 4. q : ℕ
. 5. r : ℕb
. 6. -a = q·b+r
. ⊢  ∃q:ℤ, r:ℕb. a = q·b+r  by Decide (r = 0)
. |
. |\
. | 7. r = 0
. | ⊢  ∃q:ℤ, r:ℕb. a = q·b+r  by InstConcl [-q;0]
. |
. .\
. . 7. ¬r = 0
. . ⊢  ∃q:ℤ, r:ℕb. a = q·b+r
. .   by (InstConcl [-(q+1);b-r]) THEN (ArithSimp 0)
END
```