```
dec_iff_ex_bvfun  THM

⊢  ∀E:(T→T→Prop).
|  (∀x,y:T. Dec(E(x,y))) ⟺ (∃f:(T→T→𝔹). ∀x,y:T. (x f y) ⟺ E(x,y))
| by (Unfolds [`infix_ap`;`so_apply`] 0) THEN GenRepD
|
|\
| 1. T : Type
| 2. E : T→T→Prop
| 3. ∀x,y:T. Dec(E(x,y))
| ⊢  ∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ E(x,y)
| | by (RenameVar `g` 3) THEN (Unfold `decidable` 3)
| |    THEN
| |    Witness λx,y. InjCase(g(x,y); a. true₂; b. false₂)
| |    THEN
| |    (Reduce 0)
| |
| 3. g : ∀x,y:T. E(x,y) ∨ ¬E(x,y)
| ⊢  ∀x,y:T. InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)  by RepD
| |
| 4. x : T
| 5. y : T
| ⊢  InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)  by Witness3 x
| |
| 3. g : x:T→∀y:T. E(x,y) ∨ ¬E(x,y)
| 4. x : T
| 5. y : T
| 6. y1 : ∀y:T. E(x,y) ∨ ¬E(x,y)
| 7. y1 = g(x)
| ⊢  InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)  by Witness6 y
| |
| 6. y1 : y:T→E(x,y) ∨ ¬E(x,y)
| 7. y1 = g(x)
| 8. y2 : E(x,y) ∨ ¬E(x,y)
| 9. y2 = y1(y)
| ⊢  InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)  by Analyze 8
| |
| |\
| | 8. x1 : E(x,y)
| | 9. inl(x1) = y1(y)
| | ⊢  InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)
| | | by HypSubst 7 9
| | |
| | 9. inl(x1) = g(x,y)
| | ⊢  InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)
| | | by RevHypSubst 9 0
| | |
| | ⊢  InjCase(inl(x1); a. true₂; b. false₂) ⟺ E(x,y)  by Reduce 0
| | |
| | ⊢  True ⟺ E(x,y)  by Auto
| |
| .\
| . 8. y3 : ¬E(x,y)
| . 9. inr(y3) = y1(y)
| . ⊢  InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)
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| . | by HypSubst 7 9
| . |
| . 9. inr(y3) = g(x,y)
| . ⊢  InjCase(g(x,y); a. true₂; b. false₂) ⟺ E(x,y)
| . | by (RevHypSubst 9 0) THEN (Reduce 0)
| . |
| . ⊢  False ⟺ E(x,y)  by CopyToEnd 8
| . |
| . 10. ¬E(x,y)
| . 11. E(x,y)
| . ⊢  False  by OnHyps [9;8] Thin THEN Contradiction
|
.\
. 1. T : Type
. 2. E : T→T→Prop
. 3. ∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ E(x,y)
. 4. x : T
. 5. y : T
. ⊢  Dec(E(x,y))  by Analyze 3
. |
. 3. f : T→T→𝔹
. 4. ∀x,y:T. f(x,y) ⟺ E(x,y)
. 5. x : T
. 6. y : T
. ⊢  Dec(E(x,y))  by RWO "4<" 0
. |
. ⊢  Dec(f(x,y))  by Auto
END
```