```
decidable__quotient_equal  THM

⊢  ∀E:(T→T→Prop).
|  (EquivRel x,y:T. E(x,y))
|  ⇒
|  (∀x,y:T. Dec(E(x,y))) ⇒ (∀u,v:x,y:T//E(x,y). Dec(u = v))
| by RepeatMFor 4 (Analyze 0)
|
1. T : Type
2. E : T→T→Prop
3. EquivRel x,y:T. E(x,y)
4. ∀x,y:T. Dec(E(x,y))
⊢  ∀u,v:x,y:T//E(x,y). Dec(u = v)
| by OnMCls [0;4]
|    (RWO
|     Thm* ∀E:(T→T→Prop).
|          (∀x,y:T. Dec(E(x,y)))
|          ⟺
|          (∃f:(T→T→𝔹). ∀x,y:T. (x f y) ⟺ E(x,y))
|     {dec_iff_ex_bvfun})
|
4. ∃f:(T→T→𝔹). ∀x,y:T. (x f y) ⟺ E(x,y)
⊢  ∃f:((x,y:T//E(x,y))→(x,y:T//E(x,y))→𝔹).
|  ∀u,v:x,y:T//E(x,y). (u f v) ⟺ u = v  by Analyze 4
|
4. f : T→T→𝔹
5. ∀x,y:T. (x f y) ⟺ E(x,y)
⊢  ∃f:((x,y:T//E(x,y))→(x,y:T//E(x,y))→𝔹).
|  ∀u,v:x,y:T//E(x,y). (u f v) ⟺ u = v
| by Assert (f ∈ (x,y:T//E(x,y))→(x,y:T//E(x,y))→𝔹)
|
|\
| .....assertion .....
| ⊢  f ∈ (x,y:T//E(x,y))→(x,y:T//E(x,y))→𝔹
| | by ExtWith [‘u’] [T→T→𝔹]
| |
| 6. u : x,y:T//E(x,y)
| ⊢  f(u) ∈ (x,y:T//E(x,y))→𝔹  by Analyze 6
| |
| 6. u1 : T
| 7. u2 : T
| 8. E(u1,u2)
| ⊢  f(u1) = f(u2)  by ExtWith [‘v’] [T→𝔹]
| |
| 9. v : x,y:T//E(x,y)
| ⊢  f(u1,v) = f(u2,v)  by Analyze 9
| |
| 9. v1 : T
| 10. v2 : T
| 11. E(v1,v2)
| ⊢  f(u1,v1) = f(u2,v2)
| | by BackThru
| |    Thm* ∀a,b:𝔹. (a ⟺ b) ⇒ a = b    {iff_imp_equal_bool}
| |
| ⊢  f(u1,v1) ⟺ f(u2,v2)  by RWO "5" 0
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| |
| ⊢  E(u1,v1) ⟺ E(u2,v2)  by AGenRepD ["compound";"basic"]
| |
| |\
| | 3. ∀a:T. E(a,a)
| | 4. ∀a,b:T. E(a,b) ⇒ E(b,a)
| | 5. ∀a,b,c:T. E(a,b) ⇒ E(b,c) ⇒ E(a,c)
| | 6. f : T→T→𝔹
| | 7. ∀x,y:T. (x f y) ⟺ E(x,y)
| | 8. u1 : T
| | 9. u2 : T
| | 10. E(u1,u2)
| | 11. v1 : T
| | 12. v2 : T
| | 13. E(v1,v2)
| | 14. E(u1,v1)
| | ⊢  E(u2,v2)
| |   by (FwdThru 4 [10]) THEN (FwdThru 5 [15;14])
| |      THEN
| |      (FwdThru 5 [16;13])
| |
| .\
| . 3. ∀a:T. E(a,a)
| . 4. ∀a,b:T. E(a,b) ⇒ E(b,a)
| . 5. ∀a,b,c:T. E(a,b) ⇒ E(b,c) ⇒ E(a,c)
| . 6. f : T→T→𝔹
| . 7. ∀x,y:T. (x f y) ⟺ E(x,y)
| . 8. u1 : T
| . 9. u2 : T
| . 10. E(u1,u2)
| . 11. v1 : T
| . 12. v2 : T
| . 13. E(v1,v2)
| . 14. E(u2,v2)
| . ⊢  E(u1,v1)
| .   by (FwdThru 4 [13]) THEN (FwdThru 5 [10;14])
| .      THEN
| .      (FwdThru 5 [16;15])
|
.\
. 6. f ∈ (x,y:T//E(x,y))→(x,y:T//E(x,y))→𝔹
. ⊢  ∃f:((x,y:T//E(x,y))→(x,y:T//E(x,y))→𝔹).
. |  ∀u,v:x,y:T//E(x,y). (u f v) ⟺ u = v  by Witness f
. |
. ⊢  ∀u,v:x,y:T//E(x,y). (u f v) ⟺ u = v  by RepD THEN CSquash
. |
. 7. u : x,y:T//E(x,y)
. 8. v : x,y:T//E(x,y)
. ⊢  ↓((u f v) ⟺ u = v)
. | by (UseEqWitness *) THEN (Analyze 8) THEN (Analyze 7)
. |
. 7. v1 : T
. 8. v2 : T
. 9. E(v1,v2)
. 10. u1 : T
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. 11. u2 : T
. 12. E(u1,u2)
. ⊢  * = *  by EqTypeD 0
. |
. ⊢  (u1 f v1) ⟺ u1 = v1  by GenRepD
. |
. |\
. | 13. u1 f v1
. | ⊢  u1 = v1  by (EqTypeD 0) THEN (BackThru 5)
. |
. .\
. . 13. u1 = v1
. . ⊢  u1 f v1  by (EqTypeD 13) THEN CUnhide
. . |
. . 13. E(u1,v1)
. . ⊢  u1 f v1  by BackThru 5
END
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