```
eq_rat_trans  THM

⊢  ∀x,y,z:ℚ. x =q y ⇒ y =q z ⇒ x =q z  by RepeatMFor 3 (Analyze 0)
|
1. x : ℚ
2. y : ℚ
3. z : ℚ
⊢  x =q y ⇒ y =q z ⇒ x =q z  by Unfold ‘eq_rat‘ 0
|
⊢  x.num·y.den=_2y.num·x.den
|  ⇒
|  y.num·z.den=_2z.num·y.den ⇒ x.num·z.den=_2z.num·x.den
| by RWH (LemmaC Thm* ∀x,y:ℤ. x=_2y ⟺ x = y    {assert_of_eq_int_rw})
|    0
|
⊢  x.num·y.den = y.num·x.den
|  ⇒
|  y.num·z.den = z.num·y.den ⇒ x.num·z.den = z.num·x.den
| by OnHyps [3;2;1] Analyze
|
1. x1 : ℤ
2. x2 : ℕ+
3. y1 : ℤ
4. y2 : ℕ+
5. z1 : ℤ
6. z2 : ℕ+
⊢  <x1,x2>.num·<y1,y2>.den = <y1,y2>.num·<x1,x2>.den
|  ⇒
|  <y1,y2>.num·<z1,z2>.den = <z1,z2>.num·<y1,y2>.den
|  ⇒
|  <x1,x2>.num·<z1,z2>.den = <z1,z2>.num·<x1,x2>.den
| by Unfolds [‘qnumer‘;‘qdenom‘] 0
|    THEN
|    RWH (pi1_evalC ORELSEC pi2_evalC) 0
|
⊢  x1·y2 = y1·x2 ⇒ y1·z2 = z1·y2 ⇒ x1·z2 = z1·x2  by Auto
|
7. x1·y2 = y1·x2
8. y1·z2 = z1·y2
⊢  x1·z2 = z1·x2
| by FwdThru
|    Thm* ∀i1,i2,j1,j2:ℤ. i1 = j1 ⇒ i2 = j2 ⇒ i1·i2 = j1·j2
|    {mul_functionality_wrt_eq}
|    [7;8]
|
9. x1·y2·y1·z2 = y1·x2·z1·y2
⊢  x1·z2 = z1·x2  by Decide (y1 = 0)
|
|\
| 10. y1 = 0
| ⊢  x1·z2 = z1·x2  by (HypSubst 10 8) THEN (HypSubst 10 7)
| |
| 7. x1·y2 = 0·x2
| 8. 0·z2 = z1·y2
| 9. x1·y2·y1·z2 = y1·x2·z1·y2
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| 10. y1 = 0
| ⊢  x1·z2 = z1·x2  by (ArithSimp 7) THEN (ArithSimp 8)
| |
| 7. x1·y2 = 0
| 8. 0 = y2·z1
| 9. x1·y2·y1·z2 = y1·x2·z1·y2
| 10. y1 = 0
| ⊢  x1·z2 = z1·x2
| | by FwdThru Thm* ∀a,b:ℤ. a·b = 0 ⇒ a = 0 ∨ b = 0    {int_entire}
| |    [7]
| |
| 11. x1 = 0 ∨ y2 = 0
| ⊢  x1·z2 = z1·x2
| | by InvertRel 8
| |    THEN
| |    FwdThru Thm* ∀a,b:ℤ. a·b = 0 ⇒ a = 0 ∨ b = 0    {int_entire}
| |    [8]
| |
| 8. y2·z1 = 0
| 9. x1·y2·y1·z2 = y1·x2·z1·y2
| 10. y1 = 0
| 11. x1 = 0 ∨ y2 = 0
| 12. y2 = 0 ∨ z1 = 0
| ⊢  x1·z2 = z1·x2  by Analyze 11 THEN Analyze 12
| |
| 11. x1 = 0
| 12. z1 = 0
| ⊢  x1·z2 = z1·x2  by (HypSubst 11 0) THEN (HypSubst 12 0)
|
.\
. 10. ¬y1 = 0
. ⊢  x1·z2 = z1·x2
. | by Using [‘n’,(y1·y2)]
. |    (BackThru
. |     Thm* ∀a,b:ℤ, n:ℤ⁻⁰. n·a = n·b ⇒ a = b    {mul_cancel_in_eq})
. |
. |\
. | .....set predicate .....
. | ⊢  y1·y2 ≠ 0
. |   by BackThru
. |      Thm* ∀a,b:ℤ. a ≠ 0 ⇒ b ≠ 0 ⇒ a·b ≠ 0    {int_entire_a}
. |
. .\
. . ⊢  y1·y2·x1·z2 = y1·y2·z1·x2  by Auto
END
```