```
qmul_functionality  THM

⊢  ∀p,q,r,s:ℚ. p =q q ⇒ r =q s ⇒ p *q r =q q *q s
| by RepeatMFor 4 (Analyze 0)
|
1. p : ℚ
2. q : ℚ
3. r : ℚ
4. s : ℚ
⊢  p =q q ⇒ r =q s ⇒ p *q r =q q *q s
| by Unfold ‘eq_rat‘ 0
|    THEN
|    RWH (LemmaC Thm* ∀x,y:ℤ. x=₂y ⟺ x = y    {assert_of_eq_int_rw})
|    0
|
⊢  p.num·q.den = q.num·p.den
|  ⇒
|  r.num·s.den = s.num·r.den
|  ⇒
|  p *q r.num·q *q s.den = q *q s.num·p *q r.den
| by OnHyps [4;3;2;1] RatHD
|
1. p1 : ℤ
2. p2 : ℕ⁺
3. q1 : ℤ
4. q2 : ℕ⁺
5. r1 : ℤ
6. r2 : ℕ⁺
7. s1 : ℤ
8. s2 : ℕ⁺
⊢  p1/p2.num·q1/q2.den = q1/q2.num·p1/p2.den
|  ⇒
|  r1/r2.num·s1/s2.den = s1/s2.num·r1/r2.den
|  ⇒
|  p1/p2 *q r1/r2.num·q1/q2 *q s1/s2.den
|  =
|  q1/q2 *q s1/s2.num·p1/p2 *q r1/r2.den
| by Unfold ‘qmul‘ 0 THEN RW (DepthC qnd_evalC) 0
|
⊢  p1·q2 = q1·p2 ⇒ r1·s2 = s1·r2 ⇒ p1·r1·q2·s2 = q1·s1·p2·r2
| by UnivCD
|
9. p1·q2 = q1·p2
10. r1·s2 = s1·r2
⊢  p1·r1·q2·s2 = q1·s1·p2·r2
  by FwdThru
     Thm* ∀i1,i2,j1,j2:ℤ. i1 = j1 ⇒ i2 = j2 ⇒ i1·i2 = j1·j2
     {mul_functionality_wrt_eq}
     [9;10]
END
```