```
incl_aux4_quo  THM

⊢  ∀n:{1...}, E:(ℕn→ℕn→Prop).
|  (EquivRel i,j:ℕn. i E j) ⇒
|  (∀x:i,j:ℕn//(i E j).
|   ¬x = n-1 ∈ i,j:ℕn//(i E j) ⇒ x ∈ i,j:ℕ(n-1)//(i E j))  by UnivCD
|
|\
| 1. n : {1...}
| 2. E : ℕn→ℕn→Prop
| 3. EquivRel i,j:ℕn. i E j
| 4. x : i,j:ℕn//(i E j)
| 5. ¬x = n-1
| ⊢  x ∈ i,j:ℕ(n-1)//(i E j)
| | by Inst
| |    Thm* ∀n:{1...}, m:ℕn, E:(ℕn→ℕn→Prop).
| |         (EquivRel x,y:ℕn. x E y) ⇒ (EquivRel x,y:ℕm. x E y)
| |    {rest_equi_rel}
| |    [n;n-1;E]
| |
| 6. EquivRel x,y:ℕ(n-1). x E y
| ⊢   by QuotD 4
| |
| 4. x1 : ℕn
| 5. x2 : ℕn
| 6. x1 E x2
| 7. ¬x1 = n-1
| 8. EquivRel x,y:ℕ(n-1). x E y
| ⊢  x1 = x2  by Analyze
| |
| |\
| | ⊢  x1<n-1  by Assert (¬x1 = n-1)
| | |
| | |\
| | | .....assertion .....
| | | ⊢  ¬x1 = n-1
| | |   by (Analyze 0)
| | |      THEN
| | |      (FwdThru
| | |       Thm* ∀E:(T→T→Prop).
| | |             (EquivRel x,y:T. x E y) ⇒
| | |             (∀x,y:T. x = y ⇒ x = y ∈ u,v:T//(u E v))
| | |       {incl_aux5_quo}
| | |       [3;-1])
| | |
| | .\
| | . 9. ¬x1 = n-1
| | . ⊢   by Auto
| |
| |\
| | ⊢  x2<n-1  by Assert (¬x2 = n-1)
| | |
| | |\
| | | .....assertion .....
| | | ⊢  ¬x2 = n-1
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| | | | by (Analyze 0) THEN (HypSubst -1 6) THEN (Analyze 7)
| | | |
| | | 6. x1 E (n-1)
| | | 7. EquivRel x,y:ℕ(n-1). x E y
| | | 8. x2 = n-1
| | | ⊢  x1 = n-1  by Analyze
| | |
| | .\
| | . 9. ¬x2 = n-1
| | . ⊢   by Auto
| |
| .\
| . ⊢  x1 E x2  by Trivial
|
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. 1. n : {1...}
. 2. E : ℕn→ℕn→Prop
. 3. EquivRel i,j:ℕn. i E j
. 4. i,j:ℕn//(i E j)
. ⊢  n-1 ∈ i,j:ℕn//(i E j)
.   by Inst
.      Thm* ∀T:Type, E:(T→T→Prop), x:T.
.           (EquivRel x,y:T. x E y) ⇒ x ∈ x,y:T//(x E y)
.      {incl_aux1_quo}
.      [ℕn;E;n-1]
END
```